Physics Factsheet

September 2000 Number 02

Vectors and Forces

Vector quantities are commonly encountered in physics. This Factsheet will explain:
- the difference between a vector and a scalar, giving common examples of each
- how to add and subtract vectors
- how to resolve vectors
- how to apply this to forces and particles in equilibrium

1. What is a vector?

- *A vector is a quantity that has both* ***magnitude*** *and* ***direction****.*
- *A* ***scalar*** *just has a magnitude – it is just a (positive or negative) number.*

A vector can be represented by an arrow – the length of the arrow represents the magnitude of the vector, and the direction in which the arrow is pointing shows you the vector's direction.

Anything that has a direction as well as a size will be a vector – for example, if you tell someone that your house is 200m East of the chip shop, you are representing the position of your house by a vector – its magnitude is 200m and its direction is East. If, instead, you just said that your house was 200m away from the chip shop, you are using a scalar quantity (distance). You will notice that the scalar is not so useful as the vector in telling someone where your house is!

Table 1 shows some common examples of scalar and vector quantities.

Table 1. Scalars and Vectors

Scalars	Vectors
Distance	Displacement
Speed	Velocity
Temperature	Acceleration
Energy	Force
Power	Momentum
Pressure	Torque/Moment
Mass	Impulse

Exam Hint: *Exam questions often require you to explain the difference between a scalar and a vector, and to indicate whether a particular quantity is a vector or scalar. If you think you might find it hard to work this out in an exam, make sure you learn the common examples.*

It is particularly important to understand the distance between distance and displacement, and between speed and velocity (and hence average speed and average velocity):

Suppose you walk 5m North, then 3m South. The total **distance** you have travelled is obviously 8m. However, your final **displacement** – which just means where you end up relative to where you started – is 2m North.

Suppose you were walking at a steady 2 ms^{-1}. Then your **speed** was constant throughout – it was 2 ms^{-1}. However, your **velocity** was not constant, since to begin with you were travelling at 2ms^{-1} North, then you changed to 2ms^{-1} South.

Your **average speed** for the walk is:

total distance÷total time = 8÷4 = 2ms^{-1}

(since it takes 5÷2 = 2.5s walking North and 3 ÷2 = 1.5s walking South).

However, your **average velocity** is:

total **displacement** ÷total time = 2m North ÷4 = 0.5ms^{-1} North

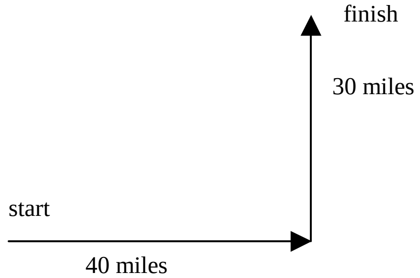
Tip: *Always be very careful not to use "speed" when you mean "velocity" and vice versa – ask yourself first whether it has a direction.*

Typical Exam Question
a) State the difference between scalar and vector quantities [1]
b) Give two examples of a vector quantity. [2]

a) a vector has direction ✓ a scalar does not
b) any two examples from vector column in table 1 ✓✓

2. Adding Vectors

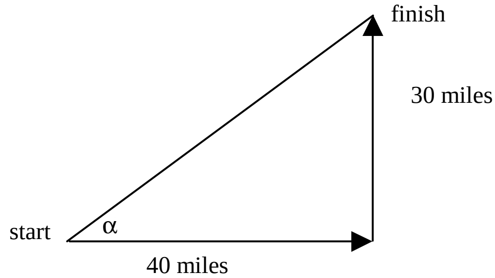
Imagine driving 40 miles East then 30 miles North. Your path would look like:



How far away from your starting point have you ended up? It certainly isn't 70 miles. If you draw the other side of the right-angled triangle, and use Pythagoras' Theorem, you will find it is 50 miles.

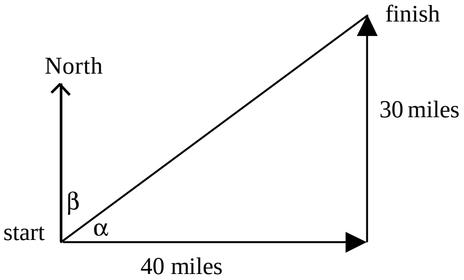
To describe the position at which you ended up, you'd say it was 50 miles from your starting point, but you'd also need to say *in what direction*. It won't be a direction like Northeast (since that would mean going the same distance North and East), so we'd have to give the direction in terms of an angle.

You might choose to find the angle marked α in the diagram below.



Using trigonometry (see the Factsheet Maths for Physics: Trigonometry if you need help on this), we find $\alpha = \tan^{-1}(0.75) = 36.9^o$.

But it is more conventional to give directions as a **bearing** – measured clockwise from North. So we'd need the angle marked β in the diagram below:



We can see that $\alpha + \beta = 90^o$, so $\beta = 53.1^o$.

We can now describe the final displacement as:
50 miles at a bearing of 53.1^o

The example above was an example of **vector addition**. We added two displacements (40 miles East and 30 miles North) to find a **resultant** diplacement (50 miles at a bearing of 53.1^o). In this case, we found the resultant by calculation (using Pythagoras' Theorem and trigonometry). It could also have been found by **scale drawing**.

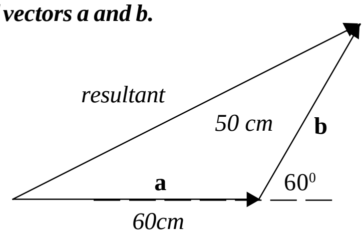
> ***Tip:*** *In any problem of this type, you MUST draw a diagram – even if it is only a sketch.*

Finding the resultant of two vectors
To find the resultant of two vectors:
- Draw one of the vectors
- Draw the other vector starting at the end of the first one.
- Draw an arrow from the start of the first vector to the end of the second one – this represents the resultant
- Use calculation or accurate measurement (if you are told to use scale drawings) to find the length (magnitude) of the resultant.
- Find the direction of the resultant

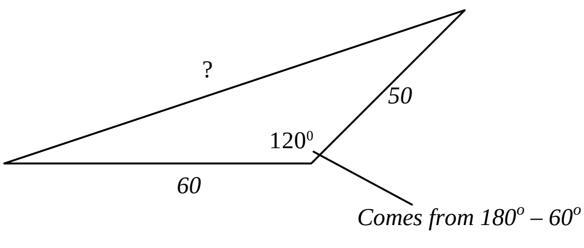
If you are using calculation, you may need to use the sine and cosine rules as well as trigonometry in normal right-angled triangles. If you do not like this sort of trigonometry, you may prefer to use the alternative method given at the end of section 3 in this Factsheet.

> ***Exam Hint:*** *If you are asked to find the resultant, you* ***must*** *give its direction as well as its length if you want full marks*

Example 1. Vector a is of magnitude 60cm and acts horizontally. Vector b is of magnitude 50cm and acts at 60^o above the horizontal. Find the resultant of vectors a and b.



First find the magnitude of the resultant; to do this we use this triangle:

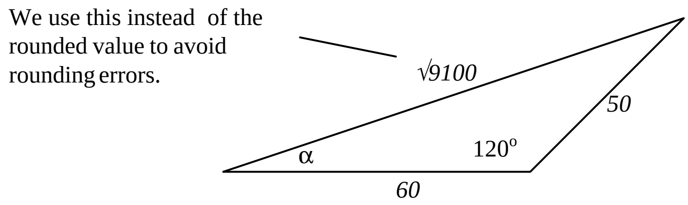


Use the cosine rule:
$a^2 = b^2 + c^2 - 2bc\cos A$; $\quad A = 120^o, a = ?, b = 60, c = 50$

$a^2 = 60^2 + 50^2 - 2 \times 60 \times 50 \times \cos 120^o$
$= 3600 + 2500 - 6000\cos 120^o$
$= 6100 + 3000 = 9100$
$a = \sqrt{9100} = 95\text{cm}$ (nearest cm)

> ***Tip:*** *When you are using the cosine rule, take care with "BIDMAS" – you must work out 2bc cosA, and subtract the answer from $b^2 + c^2$. Also, always check that your answer sounds sensible – if it does not, you may have forgotten to square root.*

Now we need to find the direction. Since in the question angles are given to the horizontal, we should give the answer in that way. So we want angle α shown below:



Use the sine rule: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$A = 120^o, a = \sqrt{9100}, b = 60, c = 50, C = \alpha$

> ***Tip****: You can write the sine rule with all the sines on the top, or with all the sines on the bottom. If you are trying to find an angle, have them on the top, and if you are trying to find a side, have them on the bottom.*

We leave out the part involving sinB, since we are not interested in it, and it is not useful. So we have:

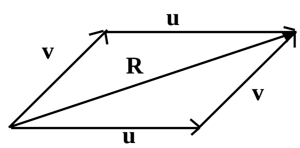
$$\frac{\sin 120}{\sqrt{9100}} = \frac{\sin\alpha}{50}$$

$$\frac{\sin 120}{\sqrt{9100}} \times 50 = \sin\alpha$$

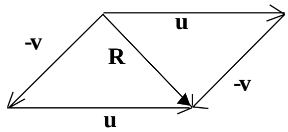
$0.45392... = \sin\alpha$
$27^o = \alpha$ (nearest degree)

So the resultant has magnitude 95cm and is at an angle of 27^o above the horizontal.

Parallelogram rule of vector addition
Let us consider adding two vectors, **u** and **v**, to give their resultant, **R**. **R** is the **diagonal** of the **parallelogram** whose sides are **u** and **v**. This gives another visual way to think about vector addition.



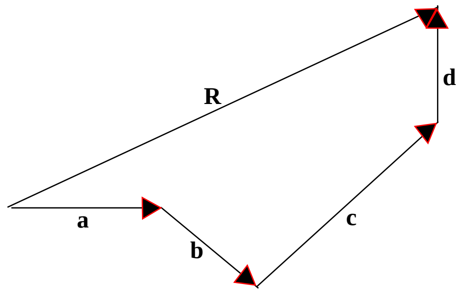
The parallelogram can also be used to **subtract** vectors - to find **u - v**, you'd draw the parallelogram using vectors **u** and **-v**:



Note that for calculation purposes, the parallelogram works exactly the same way as the vector triangle.

Resultant of more than 2 vectors

To find the resultant of more than 2 vectors, we draw a **vector polygon**. This is very similar to finding the resultant of 2 vectors, except that you draw the third vector after the second vector, and so on. The diagram shows the vector addition of vectors **a**, **b**, **c** and **d**, and their resultant, **R**.



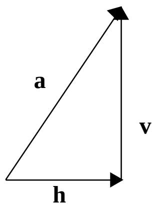
If you need to find the resultant of 3 or more vectors by calculation, it is best to use the method described at the end of section 3.

> ***Special case**: If the vector polygon **closes** (so the end of the last vector coincides with the start of the first one), then the resultant is **zero**.*

> ***Tip**: It does not matter in which order you add vectors, so if it makes your diagram or calculation easier to put them in a particular order, go ahead!*

3. Resolving

Resolving a vector involves writing it as the sum of other vectors – it's like resolving in reverse. For example, the vector **a** shown below can be written as the sum of a horizontal vector (**h**) and a vertical vector (**v**).



The seperate vectors that the original is resolved into are called **components** – in the above example, **h** is the horizontal component of **a** and **v** is the vertical component of **a**.

Note that there are many other ways we could resolve vector **a** – we choose the most convenient way in each situation (see section 4 for some examples of this). It is, however, always best to resolve a vector into two **perpendicular** components.

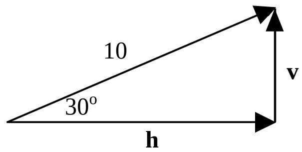
> ***Exam Hint:** In practical problems, the two directions are usually horizontally and vertically, or if an inclined plane is involved, along and perpendicular to the plane.*

Calculating components

To find the components of a vector in a pair of perpendicular directions, we will be using a right-angled triangle.

- Take the vector as the hypotenuse of the triangle
- Take the directions you want to resolve in as the other two sides
- Use trigonometry to work out the size of the components.

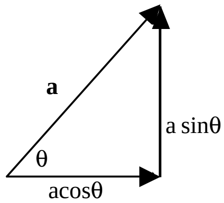
Example 2. A vector of magnitude 10 is inclined at 30° above the horizontal. Find the horizontal and vertical components of the vector.



Using trigonometry: $\cos 30^o = \frac{h}{10}$, so $h = 10 \times \cos 30^o = 8.66$

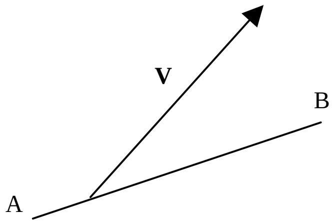
$\sin 30^o = \frac{v}{10}$, so $v = 10 \times \sin 30^o = 5$

In fact, you can save time working out the trigonometry by remembering the following diagram showing the vector **a** and its components:



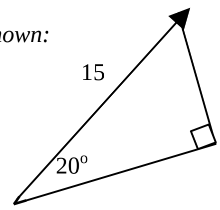
The following example shows how to apply this.

Example 3. The diagram below shows a vector V and the line AB.

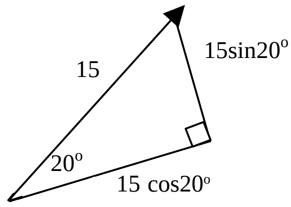


The angle between V and AB is 20°. Find the components of V parallel and perpendicular to AB, given that the magnitude of V is 15.

Our triangle is as shown:



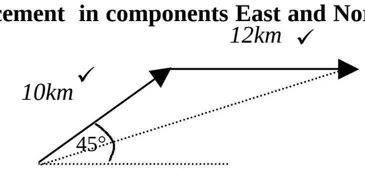
By comparison with the diagram above, we get:



So the component parallel to AB is 15cos20° = 14 (2 SF)
The component perpendicular to AB is 15sin20° = 5.1 (2SF)

Typical Exam Question
Bill travels 10km North-east and then 12km due East
(a) Draw a vector diagram showing Bill's route. [2]
(b) Calculate, without the use of a scale diagram, Bill's resultant displacement in components East and North. [3]

(a)



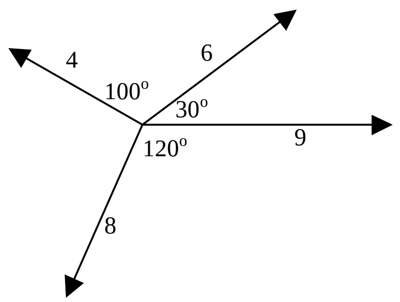
(b) E: 10cos45 ✓+ 12✓ = 19km. N: 10sin45 = 7.1km✓

Using components to find the resultant
For problems involving many vectors, the following provides a fail-safe method to find the resultant force:

- Choose two sensible perpendicular directions – horizontal and vertical are often a good idea.
- Resolve every vector involved in these two directions (you could put them in a table to aid clarity and ensure you have not missed one out)
- Add up all the components in one direction (say horizontal), which gives the resultant force in that direction.
- Repeat for all the components in the other direction.
- You are now left with two perpendicular vectors. Find the resultant of these two vectors using Pythagoras and basic trigonometry.

> ***Exam Hint**: A common mistake is to resolve correctly, but ignore the direction of the component – for example, 6 units upwards is not the same as 6 units downwards! To avoid confusion, decide at the beginning which direction to take as positive.*

Example 4. Find the resultant of the vectors shown below.



We will take to the left, and upwards as positive

Vector	*Horizontal compt*	*Vertical compt*
9	9	0
6	6 cos30°	6sin30°
4	-4cos50°	4sin50°
8	-8cos60°	-8sin60°

The angles 50° and 60° come from using angles on a straight line = 180°

> ***Tip:** Do not actually work out the sines and cosines yet, to avoid rounding errors or copying errors.*

So total of horizontal components $= 9 + 6\cos30^o - 4\cos50^o - 8\cos60^o$
$= 7.625...$
total of vertical components $= 0 + 6\sin30^o + 4\sin50^o - 8\sin60^o$
$= -1.864...$

So to find overall resultant:
By Pythagoras, magnitude of $R = \sqrt{(7.625^2 + 1.864^2)} = 7.8$ *(2 SF)*
$\alpha = \tan^{-1}(1.864 \div 7.625) = 14^o$ *below the horizontal (2 SF)*

4. Application to forces and equilibrium
To find the resultant of a number of forces, use the methods described above.

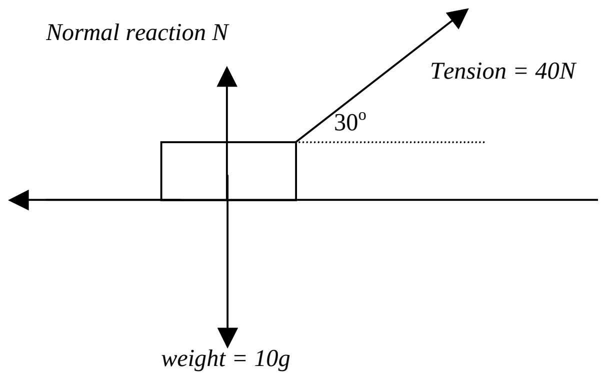
A body is in **equilibrium** if there is no resultant force (and no resultant torque – see Factsheet 4 Moments and Equilibrium) on it. In the examples considered in this Factsheet, there will never be a resultant torque, so we will only have to use that the resultant force is zero.

Any body is in equilibrium if it is at rest, or moving with constant velocity (**not** just a constant speed) – see Factsheet 12 Applying Newton's Laws for more details on this.

Problems involving equilibrium often require you to use the fact that the body is in equilibrium to find unknown forces. The procedure here is:
- First read the question carefully to check whether the particle is in equilibrium
- Draw a diagram showing all the forces
- Resolve all forces in two perpendicular directions
- Find the total of the components in each direction, and equate it to 0.
- Solve your equations to find any unknown forces.

Example 5. A box of mass 10kg is being towed at constant velocity along rough horizontal ground by a rope inclined at 30° to the horizontal. The tension in the rope is 40N. Find:
a) the frictional force exerted by the ground on the box
b) the normal reaction force exerted by the ground on the box.
Take g = 9.8ms⁻²

We know the box is in equilibrium because it is moving with ***constant velocity***

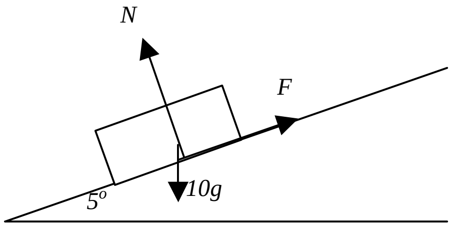


Resolve horizontally and vertically (taking upwards and left as positive), and equate to 0:

$\rightarrow$ $40\cos30^o - F = 0$
$\uparrow$ $40\sin30^o + N - 10g = 0$

From the first equation, we get $F = 40\cos30^o = 35N$ *(2 SF)*
From the second equation, we get $N = 10g - 40\sin30^o = 78N$

Example 6. A box of mass 10kg is at rest on a rough plane inclined at 5° to the horizontal. Find the normal reaction and the frictional force exerted by the plane on the box. Take $g = 9.8\ ms^{-2}$



The box is in equilibrium as it is at rest.
Note that friction must act up the plane, since the box will be "trying" to slide down.

When there is an inclined plane involved, it is best to resolve parallel and perpendicular to the plane.

The weight of the box acts at an angle of 85° to the plane (from angles in a triangle). So we have:

Along the plane: $F - 10g\cos85^o = 0 \Rightarrow F = 8.5N$ *(2SF)*
Perpendicular to the plane: $N - 10g\sin85^o = 0 \Rightarrow N = 98N$ *(2SF)*

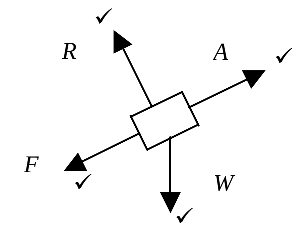
Typical Exam Question

A body of mass 5.0kg is pulled up a rough plane, which is inclined at 30° to the horizontal, by the application of a constant force of 50N, which acts parallel to the plane. Take *g = 9.8 ms⁻²*

(a) Draw a free body diagram for the body showing the normal reaction R, frictional force F, weight W and the applied force A.[4]

(b) When the arrangement is in equilibrium, what are the values of the normal reaction force R and the frictional force F? [4]

(a)



(b) Resolve forces perpendicular to the plane:

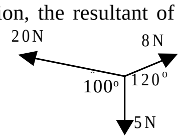
R = Wcos30° ✓ so R = 42.4 ≈42N ✓

Resolve forces parallel to the plane:

F + Wsin30° = 50 ✓ so F= 21.7 ≈ 22N ✓

Questions

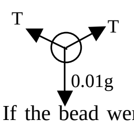
1. a) Explain the difference between a vector and a scalar
   b) Indicate whether each of the following is a scalar or a vector:
      Density Momentum Electrical resistance Distance Acceleration
2. Find the resultant of each of the following:
   a) A force of 5N acting horizontally to the left and a force of 8N acting vertically upwards
   b) A force of 10N acting vertically upwards and a force of 3N acting at 20° to the upward vertical.
   c) A force of 5N acting at 10° below the horizontal and a force of 5N acting at 10° above the horizontal
   d) A force of 6N acting horizontally to the left, a force of 8N acting vertically upwards and a force of 2N acting horizontally to the right.
3. A body is in equilibrium. Which of the following **must** be true? There may be more than one correct answer.
   a) the body is stationary
   b) the polygon of forces for the body is closed
   c) there are no forces acting on the body.
4. Find, by calculation, the resultant of the forces shown below :



5. A smooth bead of mass 10g is threaded onto a thin piece of string of length 2m. The ends of the string are fastened to the ceiling so that they are at the same horizontal level as each other and are 1.6m apart.
   a) Draw a diagram to show the forces acting on the bead.
   b) Explain why the bead only rests in equilibrium at the midpoint of the string.
   c) Find the tension in the string, taking g = 10ms⁻²

Answers

1. a) A vector has magnitude and direction; a scalar has magnitude only
   b) scalar, vector, scalar, scalar, vector
2. a) magnitude 9.4N, at 58° above horizontal
   b) magnitude 13N, at 4.6° to upward vertical
   c) 9.8N acting horizontally
   d) 8.9N acting at 63° above (leftward) horizontal
3. b) only
4. 13N at 11° above rightward horizontal
5. a)



   b) If the bead were not at the midpoint, the angles made by the two pieces of string would not be equal. If the angles were not equal, there would be a net horizontal force on the bead,since the horizontal components of the tensions would not balance, so the bead would not be in equilibrium.
   c) 0.083N (2SF)

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A boat travels at 6.8ms⁻¹ South-easterly towards a harbour, which is 10km away. Once the boat reaches harbour, the passengers get in a car and drive due North at 80kmhr⁻¹ for 15 minutes. Calculate the:

(a) total distance travelled. [2]

d = s × t = 80 × ¼ = 20 + 10 = 30 ✓

1/2

> 1 mark deducted for omission of units.
> Although it would not lose marks, the candidate should avoid writing 80 × ¼ =...= 30, since it is not mathematically correct and could lead to confusion

(b) total displacement. [5]

20² – 10² = 300.

√300 = 17.3 km

0/5

> Candidate has attempted to treat this as a right-angled triangle, when it is not – either the cosine rule or resolving into components should have been used. Candidate has also not given the direction

(c) average speed. [2]

80kmh⁻¹ = 80 × 1000/3600 = 22.2ms⁻¹

So average speed = (22.2 + 6.8) ÷ 2 = 14.5ms⁻¹

0/2

> Candidate has not used the correct definition of average speed – it is total distance/ total time, **not** the average of the individual speeds.

(d) magnitude of the average velocity. [2]

14.5ms⁻¹

0/2

> Poor exam technique – the candidate should appreciate that 2 marks will not be awarded for simply writing down the same answer again. Although the candidate's answer for total displacement was incorrect, credit would have been awarded if it had been used correctly to find average velocity.

Examiner's Answers

(a) Total distance = 10 + (80 × ¼) ✓ = 30km ✓

(b) Total displacement = 10km SE + 20km N

East = 10 cos45

North = -10 sin45 + 20.

= (7.1, 12.9) km ✓

Magnitude is |R|= √(7.1² + 12.9²) ✓ =14.7 km

Direction is given by θ = tan⁻¹(12.9 / 7.1) ✓ = 61.2°

Displacement vector = 15km, ✓ 61° N of E (or bearing 029) ✓

(c) Average speed = total distance / total time

= 30 km/ (25 + 15) minutes ✓

= 0.75 km / minute = 45 km / hr ✓

(d) Magnitude of average velocity

= (magnitude of total displacement) / total time

= (14.7 km)/ (⅔ hr) ✓ = 22 km / hr ✓

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