Stress, strain & young modulus

Stress

Stress is a measure of the internal force an object is experiencing per unit cross sectional area:

Where σ is stress (in Newtons per square metre or, equivalently, Pascals), F is force (in Newtons, commonly abbreviated N), and A is the cross sectional area of the sample.

This is identical to the formula for pressure. Two distinctions should be made between stress and pressure: Firstly, while pressure is typically used to describe fluids (liquids or gases), stress is used to describe solids. Secondly, while pressure can only act perpendicular to a surface, stress is also able to act parallel to a surface. Stresses acting parallel to a surface are known as shear stresses.

Tensile Strength

The (ultimate) tensile strength is the level of stress at which a material will fracture. Tensile strength is also known as fracture stress. If a material fractures by 'crack propagation' (i.e., it shatters), the material is brittle.

Yield Stress

On a stress strain graph beyond the yield point (or elastic limit) the material will no longer return to its original length. This means it has become permanently deformed. Therefore the yield stress is the level of stress at which a material will deform permanently. This is also known as yield strength.

Strain

Stresses lead to strain (or deformation). Putting pressure on an object causes it to stretch. Strain is a measure of how much an object is being stretched. The formula for strain is:

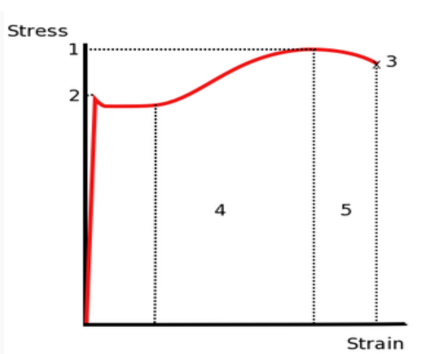
where is the original length of a bar being stretched, and l is its length after it has been stretched. Δl is the extension of the bar, the difference between these two lengths.

Young's Modulus

Young's Modulus is a measure of the stiffness of a material, and describes how much strain a material will undergo (i.e. how much it will stretch) as a result of a given amount of stress. The Young's Modulus E of a material is calculated as:

The values for stress and strain must be taken at as low a stress level as possible, provided a difference in the length of the sample can be measured. Strain has no units due to simply being the ratio between the extension and original length of a material, so Young's Modulus is measured by the same units as stress, i.e. newtons per square metre (Nm^{-2}) or Pascals (Pa). As Young's Modulus is measured per unit area, it is an intensive property, meaning it only depends on the material being used, and not on the size of the material.

Stress-Strain Graphs



Stress–strain curve for low-carbon steel.

Stress, strain & young modulus

Stress (σ) can be graphed against strain (ε). The toughness of a material (i.e., how much it resists stress, in J m^{-3}) is equal to the area under the curve, between the y-axis and the fracture point. Graphs such as the one on the right show how stress affects a material. This image shows the stress-strain graph for low-carbon steel. It has three main features:

Elastic Region

In this region (between the origin and point 2), the ratio of stress to strain (Young's modulus) is constant, meaning that the material is obeying Hooke's law, which states that a material is elastic (it will return to its original shape) if force is directly proportional to extension of the material

Hooke's Law

Hooke's law of elasticity is an approximation that states that the Force F (load) is in direct proportion with the extension (denoted by x or e) of a material, provided this load does not exceed the proportional limit. The constant k is inherent to the material being measured. Materials for which Hooke's law is a useful approximation are known as linear-elastic, or 'elastic materials'.

Plastic Region

In this region (between points 2 and 3), the rate at which extension is increasing is going up, and the material has passed the elastic limit - it will no longer return to its original shape when the load is removed, and will no undergo plastic (permanent) deformation. After point 1, the amount of stress decreases due to necking at one point in the specimen. If the stress were recorded where the necking occurs we would observe an upward curve and an increase in stress due to this reduction in area(stress = Force / area, thus stress increases during necking). The material will now 'give' and extend more under less force.

Fracture Point

At point 3, the material has been fractured and so no further measurements can be taken.

Other Typical Graphs

In a brittle material, such as glass or ceramics, the stress-strain graph will have an extremely short elastic region, and then will fracture. There is no plastic region on the stress-strain graph of a brittle material.

Questions

1) 100N of force are exerted on a wire with cross-sectional area 0.50mm^2. How much stress is being exerted on the wire?
2) Another wire has a tensile strength of 70MPa, and breaks under 100N of force. What is the cross-sectional area of the wire just before breaking?
3) What is the strain on a Twix bar (original length 10 cm) if it is now 12 cm long?
4) What is this strain, expressed as a percentage?
5) 50N are applied to a wire with a radius of 1mm. The wire was 0.7m long, but is now 0.75m long. What is the Young's Modulus for the material the wire is made of?
6) Glass, a brittle material, fractures at a strain of 0.004 and a stress of 240 MPa. Sketch the stress-strain graph for glass.
7) (Extra nasty question which you won't ever get in an exam) What is the toughness of glass?
8) Wire has a tensile strength of 0.95Mpa, and breaks under 25N of force. what is the cross-sectional area of the wire before and after breaking?