Physics Factsheet

www.curriculum-press.co.uk Number 100

Stationary Waves on Strings and in Air Columns

This factsheet will:
- **Explain how stationary (standing) waves are formed**
- **Consider the different modes of vibration and the effect of this on musical notes**
- **Consider the similarities and differences between stationary waves on strings (eg guitar) and those in air columns (eg organ pipes)**
- **Give you practice and guidance on doing exam-style questions**

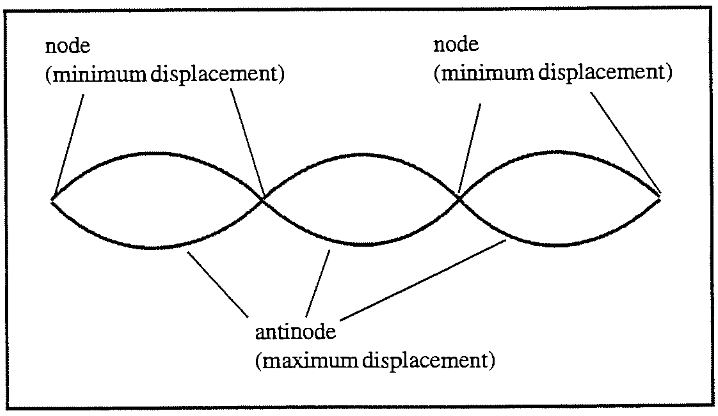
A stationary wave is formed when two waves travelling in opposite directions interfere. To make a stationary wave, the two waves must have:
- the same **speed**
- the same **frequency**
- equal or nearly equal **amplitudes**

> **One easy way for this to happen is for a wave to interfere with its own reflection.** This is what happens in stringed and woodwind instruments. Before we look at strings and air columns, though, here are some general points about stationary waves.

There are points where the displacement is always zero – these are called **nodes**.

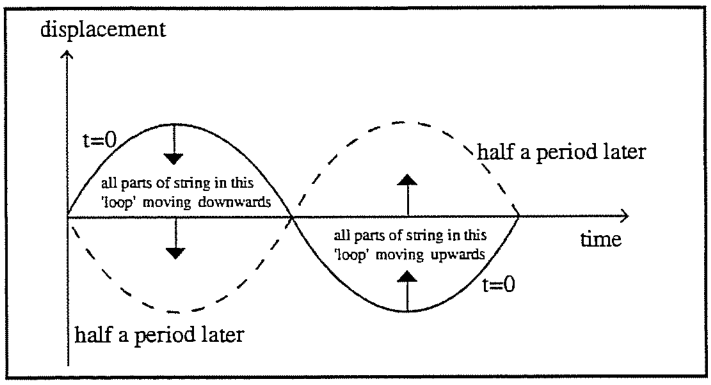
Midway between the nodes are **antinodes**, where the displacement is greatest (Fig 1).

Fig 1



At any one time all the particles in one "loop" are **in phase**, and each loop is exactly out of phase with adjacent loops (Fig 2).

Fig 2



Example 1
A stationary wave is set up on a wire of length 0.93 m so that it vibrates at 120 Hz. The fundamental frequency of the wire is 40 Hz.
(a) Draw a sketch of the stationary wave obtained. [1 mark]
(b) Calculate the speed of the wave. [3 marks]

Answer
The frequency is 3 times the fundamental frequency, so we have the third harmonic

[1]

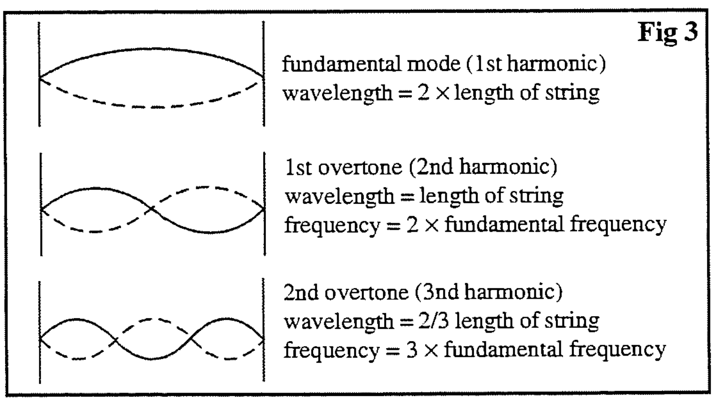
The wavelength is the length of two loops
= 2/3 × 0.93 m = 0.62 m *[1]*

For the speed, $v = f\lambda$
$= 120\ Hz \times 0.62\ m$ *[1]*
$= 74\ m\ s^{-1}$ *[1]*

Stationary Waves on Strings

When one end of a stretched string is vibrated, a travelling wave moves along the string, and reflects from the other end. *The wave interferes with its own reflection* and so a stationary wave is set up. The ends of the string *can not move*, and so must be **nodes**. This one fact allows us to draw all the possible modes of vibration.

Fig 3

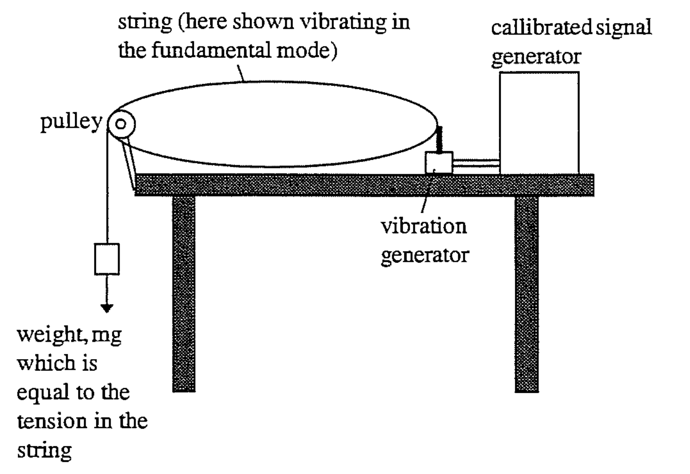


Notice that two adjacent nodes (or antinodes) are half a wavelength apart.

> ***Exam Hint***
> *You can always find the wavelength if you know the length of the string:*
> - *the wavelength is the "length of two loops".*
> - *So for the ninth harmonic, which has nine loops, the wavelength is two ninths of the length of the string*

You can *force* the string to vibrate in a given mode by using a vibration generator oscillating at the required frequency Fig 4.

Fig 4



However, if simply plucked, the string will vibrate in all modes at once. The **fundamental mode**, will, in general have the largest amplitude, and the frequency of the fundamental determines the **pitch** obtained. **Superposed** on the fundamental are the other **harmonics** in varying 'strengths' – it is the relative amplitudes of these other frequencies that make middle 'C' on a guitar, say, sound different from middle 'C' on a piano. The quality of a sound, due to 2nd and higher harmonics is sometimes called its **timbre**.

The profile of a stationary wave does not move along the wire, but the two travelling waves that produce it have a speed v equal to:

> $v = \sqrt{\frac{T}{\mu}}$ where T is the **tension** in the string, and μ is its **mass per unit length**

Since $v = f\lambda$ and the **wavelength** of the fundamental mode is $2L$, then the **fundamental frequency** f_0 for a stretched string is given by

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

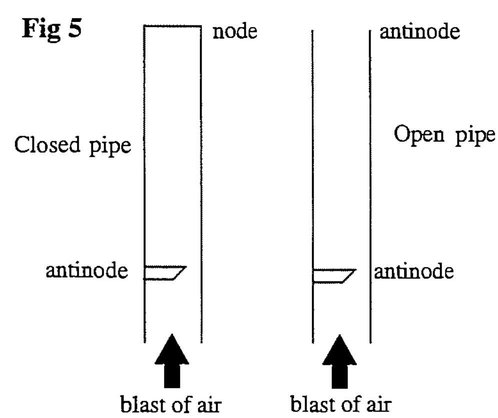
Notice that **length**, **tension** and **mass per unit length** are the only factors affecting the pitch of the note. You knew this already: to make a guitar play a higher pitch, you:
(a) shorten the string by holding a string down
(b) tighten the string
(c) play a thinner string

> ***Example 2***
> ***A signal generator is set at 152 Hz, 10 loops fit the length of the vibrating length of string exactly. The string is of length 2.0 m and the mass on the end of it is 0.72 kg. Calculate the mass of the string.*** **[5 marks]**
>
> **Answer**
> *The wavelength is the length of two loops = 2/10 x 2.0 m = 0.4 m*
>
> *For the speed,* $v = f\lambda$
> $= 152 \text{ Hz} \times 0.4 \text{ m}$
> $= 60.8 \text{ m s}^{-1}$ *[1]*
>
> *We can now use this in*
> $$v = \sqrt{\frac{T}{\mu}}$$
> $$\Rightarrow \mu = \frac{T}{v^2}$$ *[1]*
> *(check you can rearrange this correctly)*
>
> *The tension, T, is equal to the weight hanging from it*
> $= 0.72 \text{ kg} \times 9.8 \text{ N kg}^{-1} = 7.056 \text{ N}$ *[1]*
>
> *Therefore,* $\mu = \frac{7.056}{60.8^2}$
> $= 1.9 \times 10^{-3} \text{ kg m}^{-1}$ (1.9 g m^{-1}) *[1]*
>
> *And for the 2.0 m the mass is therefore 3.8 g* [1]

Stationary Waves in Air Columns

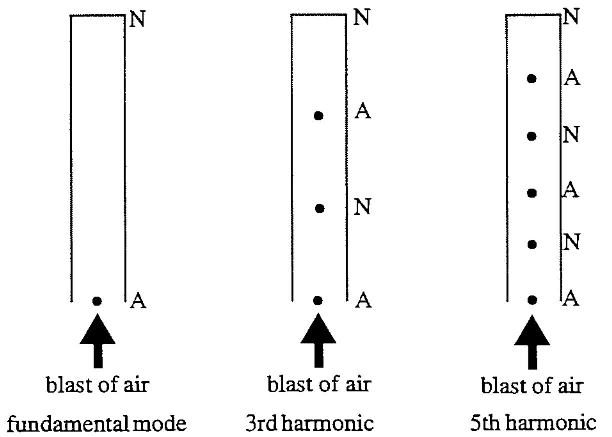
These are set up when the air at one end of a pipe is caused to vibrate. (In an organ this is done by blasting air at an edge). The resulting sound wave *interferes with its own reflection* from the other end of the pipe to produce the stationary wave.

These are very similar to stationary waves on strings. One important difference is that the stationary waves are longitudinal rather than transverse because they are formed from sound waves.

> **Open pipes** and **closed pipes** behave slightly differently.
> - In both cases at the end where the air is blasted in, the air is free to move, and so there is an antinode.
> - In a closed pipe, the closed end prevents air moving and so there must be a node here.
> - In an open pipe, the sound wave reflects off the free air at the end of the pipe, and since the air is free to move here, then there is an antinode at the open end.
>
> **Fig 5**
>
> 
>

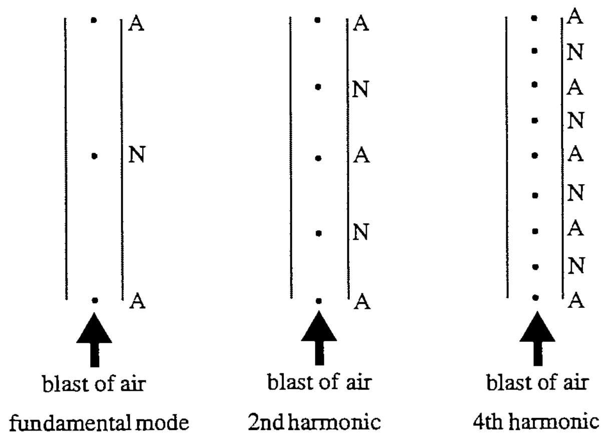
Because of this, the modes of vibration for open and closed pipes are different Fig 6.

Fig 6



Because there must be an antinode at the open end and a node at the closed end, a closed pipe can only produce odd harmonics

Fig 7



Notice that, because there is an antinode at each end, an open pipe can produce both odd and even harmonics. This gives it a richer sound than a closed pipe.

Notice also that the wavelength of the fundamental mode in the open pipe is half that of the closed pipe, and thus the fundamental frequency of an open pipe is twice that of a closed pipe.

Practice Questions

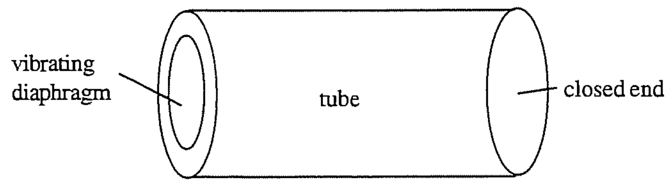
1. A horizontal string of length 0.65 m has a mass of 5.5 g and is put under a tension of 110 N. It is plucked. (a) Calculate the speed of the transverse waves on the string. (b) Calculate the wavelengths and frequencies of the fundamental and second harmonic.

2. The fundamental frequency of vibration of a stretched wire is 120 Hz. Calculate the new fundamental frequency if (a) the tension in the wire is doubled, the length remaining constant, (b) the length is doubled with constant tension, (c) the tension in the wire is doubled and the length of the wire is also doubled.

3. A guitar string is 0.70 m long. The string is tuned so that when its full length is plucked it vibrates at a frequency of 384 Hz. To play a higher note, the string is pressed so that the length free to vibrate is shorter. A fret (ridge) on the neck of the guitar ensures that the correct length is produced when the string is pressed. A certain fret is positioned so that when it is used, the frequency of the note obtained is 427 Hz. What length of string is vibrating now?

4. What is the frequency of the sound emitted by an open-ended organ pipe 1.7 m long when sounding its fundamental frequency, if the speed of sound in air is 340 m s^{-1}?

5.



In the diagram above, when the diaphragm vibrates at 2000 Hz a stationary wave pattern is set up, and the distance between adjacent nodes is 8.0 cm. When the frequency is gradually reduced, the stationary wave pattern disappears, but then reappears at a frequency of 1600 Hz. Calculate:
(a) the speed of sound in air [2 marks]
(b) the distance between adjacent nodes at 1600 Hz [2 marks]
(c) the next lower frequency at which a stationary wave is obtained [1 mark]
(d) the length of the tube [2 marks]

6. A piece of glass tubing is closed at one end by covering it with a sheet of metal. The fundamental frequency is found to be 280 Hz. Calculate the length of the tube. If the metal sheet is now removed, calculate the wavelengths and frequencies of the fundamental and second harmonic of the resulting open pipe.

Answers

1. 114 ms^{-1},
fundamental: wavelength = 1.30 m, f = 87.7 Hz
second harmonic: wavelength = 0.65 m, f = 175 Hz

2. 170 Hz, 60 Hz, 84.9 Hz

3. 0.63 m

4. 100 Hz

5. (a) 320 m s^{-1}
(b) 0.10 m apart
(c) 1200 Hz
(d) wavelength is 0.80 m, the tube is 0.40 m long

6. 0.304 m
0.607 m, 560 Hz. 0.304 m, 1120 Hz

Acknowledgements:
This Physics Factsheet was researched and written by Michael Lingard
The Curriculum Press, Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU

ISSN 1351-5136