Physics Factsheet

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Energy in Electric Circuits

The purpose of this Factsheet is to bring together ideas of energy transfer in electric circuits. Before studying the Factsheet you should make sure that you are familiar with the ideas of types of energy and ideas of current, potential difference, and resistance from your GCSE course.

You will also find the following Factsheets useful to consolidate some ideas:
Factsheet 5: for ideas of work energy and power.
Factsheet 7: for ideas of current, voltage and resistance.
Factsheet 23: for ideas of currents in circuits, internal resistance and Kirchhoff's Laws.
Factsheet 31: which defines working and heating, for thermodynamics considerations.

In questions at AS Level, you are likely to find that you are required to use ideas of energy transfer combined with other ideas such as internal resistance. In the synoptic paper at A2, you are likely to find the ideas combined with the concept of heating for thermodynamics questions.

Energy transfer in the cell.
At GCSE level you learned that in a cell, chemical energy is transformed into electrical energy. At A level you should recognise this process as working, as opposed to heating, because it is an ordered process and not due to a temperature difference.

The e.m.f., ε, of the cell is defined to be the energy transformed in moving a unit charge across the cell between the plates. So the e.m.f, ε (in Volts) is the work done (in Joules) per unit charge (in coulombs).

$$\varepsilon = \frac{\Delta W}{q}$$

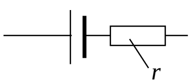
where: ε = *e.m.f of the cell (V)*
ΔW = *work done (J)*
q = *unit charge (Q)*

Worked Example

1. (a) What is the energy change when a charge of 5μC is transferred across a cell of e.m.f. 3V?
Energy change = e.m.f × charge = $3 \times 5 \times 10^{-6} = 15\mu$ *J*

(b) 9μJ of work is done when a charge of 6μC is moved across a cell. Wwhat is its e.m.f?
$$\text{e.m.f} = \frac{9 \times 10^{-6}\ J}{6 \times 10^{-6} C} = \frac{1.5 J}{C} = 1.5V$$

If the e.m.f drives a current, I, then I coulombs of charge are moved across the cell per second, and if the current continues for t seconds, then the work done = $\varepsilon \times I \times t$. This is the chemical energy transformed, however, the cell offers some resistance to the flow of charge, so not all of the energy is transformed into electrical energy. The potential difference appearing across the terminals of the cell (the terminal P.D. V) is less than the e.m.f. and the difference is described as the "lost volts". The cell is usually drawn as if it had a resistor in series with it, though the resistance is actually within the body of the cell. This resistance is described as the "internal resistance" (r) of the cell.



The "lost volts" depends on the current drawn from the cell, since the lost volts will be $I \times r$. This leads to the equation:

$$V = \varepsilon - Ir$$

where V = terminal p.d (Volts)
ε = e.m.f of cell (Volts)
I = curremt (amps)
r = internal resistance (Ω)

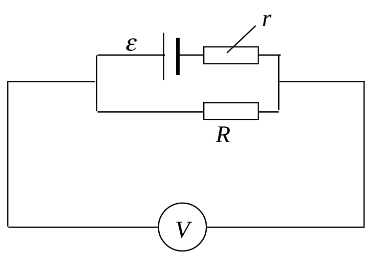
Energy transfer in a load resistor
The P.D, V across a resistor gives the energy transfer per unit charge. The current, I, through the resistor gives the number of coulombs of charge per s, so $V \times I$ gives the energy change per second. – the **power**. Combining this with Ohm's Law $V = I \times R$ gives $P = I \times R \times R = I^2R$. This is the energy per s dissipated as heat in the resistor.

$$P = I \times R \times R = I^2R$$

Energy wasted by the internal resistance
Applying these ideas to the internal resistance gives the energy per second wasted by the internal resistance as I^2r.

Energy wasted per second by the internal resistance = I^2r

Energy transfered in the circuit



In the circuit shown, an external resistor, R is connected across a cell of e.m.f., ε and internal resistance, r. The voltmeter measures the P.D. across the resistor, which is also the terminal P.D. of the cell.

The power dissipated in the resistor (energy per second) is $V \times I$. If I is large, V will be small, because drawing more current from the cell increases the lost volts. For V to be large, I will be small, so to get maximum power from the resistor, a compromise is needed. The value of R to give maximum power in the load resistor can be calculated. Students also doing A Level maths might like to do this calculation by getting an expression for the power ($V \times I$) in the load in terms of ε, R and r, differentiating it, and setting the differential to zero. [Don't worry if you have not done enough maths to do this.] It turns out that the maximum power delivered is when R = r; the external load resistance is the same as the internal resistance of the cell.

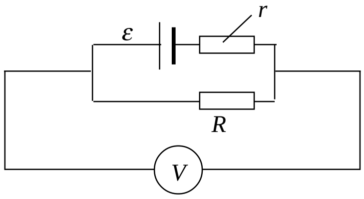
Kirchhoffs 2nd Law

Factsheet 23 deals with Kirchhoff's laws in detail. You should appreciate that Kirchhoff's 2nd Law is really a statement of conservation of energy. The $\Sigma\varepsilon$ is the sum of the energy being transformed into electrical energy by the cells and the ΣIR is the sum of the electrical energy being dissipated as heat by the resistors. One being taken as positive and the other negative means that we can write: $\Sigma\varepsilon = \Sigma IR$

> *Kirchhoff's 2nd Law: $\Sigma\varepsilon = \Sigma IR$, is a statement of conservation of energy*

Practice Questions

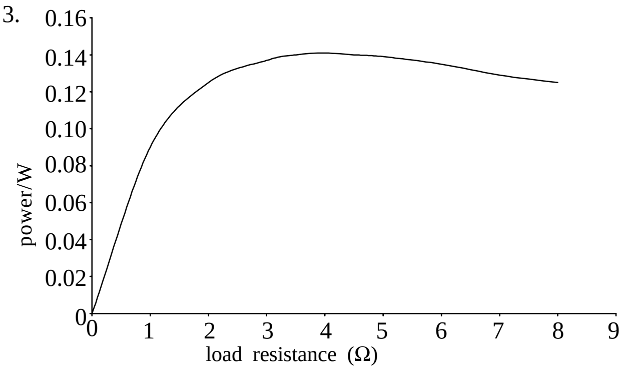
1. Define the e.m.f of a cell.
2. (a) What is the energy transfer per second for a cell of e.m.f. 6V when it is delivering a current of 0.2A?
   (b) If it has internal resistance of 0.3Ω, how much energy is wasted per second?
   (c) What will be the terminal P.D. of the cell, in these conditions?
   (d) What will be its terminal P.D. when it is delivering a current of 0.5A?
3. A resistor is used in the circuit shown.



(a) If the value of r is 4Ω, and ε = 1.5V, calculate values of the power dissipated in the load for values of R between 0 and 8Ω and draw up a table.
(b) Plot power against R, including more values around R = 4Ω as appropriate, so that you can draw a representative curve.
(c) Hence show that the maximum value of the power is when R = r

Answers

1. The e.m.f of a cell is the work done in moving a unit charge across the cell.
2. (a) Energy transfer 6J/C × 0.2C/s = 1.2J/s
   (b) Wasted energy = I^2r = 0.2 × 0.2 x 0.3 = 1.2×10^{-3}J
   (c) V = ε - Ir = 6 – (0.2 × 0.3) = 5.94V
   (d) V = ε - Ir = 6 – (0.5 × 0.3) = 5.85V
3.



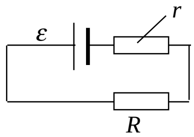
Use $I = \dfrac{\varepsilon}{R+r} = \dfrac{1.5}{R+4}$

R	0	1	2	3	4	5	6	7	8	3.5	4.5
R+4	4	5	6	7	8	9	10	11	12	7.5	8.5
I	0.375	0.3	0.25	0.214	0.188	0.167	0.15	0.136	0.125	0.2	0.1765
I^2R	0	0.09	0.125	0.137	0.141	0.139	0.135	0.129	0.125	0.14	0.14

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they could be improved. The examiner's mark scheme is given below.

The diagram shows a cell, of e.m.f ε and internal resistance, r, driving a current I through a load resistor as shown.



(a) Using these symbols, write down a formula for
(i) the power dissipated in the load resistor. (1)
power = V × I *0/1*

> The candidate has merely written down the standard formula, and not used the symbols given in the question, *ε*, *I*, *R* and *r*

(ii) the power dissipated by the internal resistance (1)
power = V × I *0/1*

> Again the candidate has failed to apply a formula to the context of the question.

(iii) the rate of conversion of energy in the cell (1)
Energy = ε *0/1*

> The candidate knows that energy conversion has something to do with the e.m.f. but not what.

(b) Using these formulae, write down an equation for the conservation of energy in the circuit, and hence show that

$$I = \frac{\varepsilon}{(R + r)}$$ **(2)**

0/2

> Since the candidate has failed to obtain the correct expressions, s/he is unable to put them together.

Examiner's answers

(a) (i) I^2R ✓
(ii) I^2r ✓
(iii) $\varepsilon \times I$ ✓

(b) Conservation of energy requires that the energy per second transformed in the cell ε x I equals the energy transformed per second in the circuit, so
$\varepsilon \times I = I^2R + I^2r$ ✓
dividing by I gives
$\varepsilon = I(R + r)$ *so* ✓
$I = \dfrac{\varepsilon}{(R + r)}$ ✓

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