Physics Factsheet

April 2003 Number 52

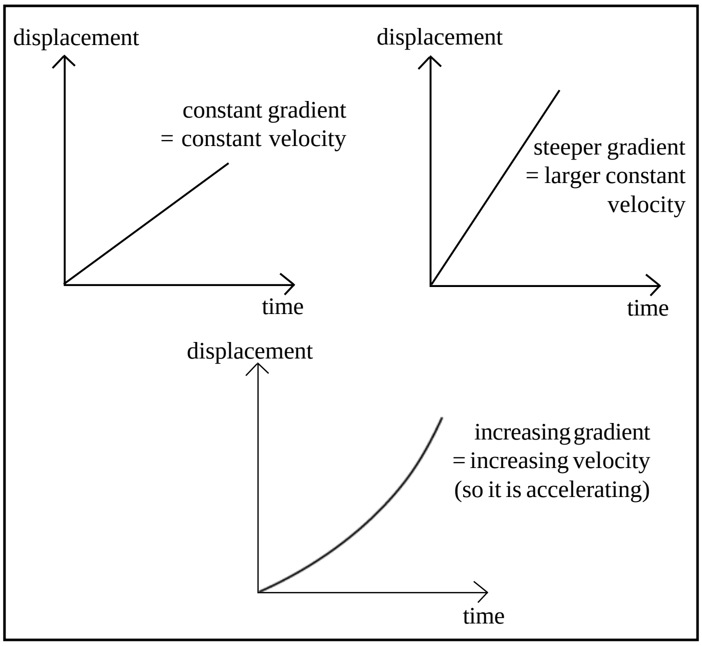
# Displacement-time and Velocity-time Graphs

This Factsheet explains how motion can be described using graphs, in particular how displacement-time graphs and velocity-time graphs can be used.

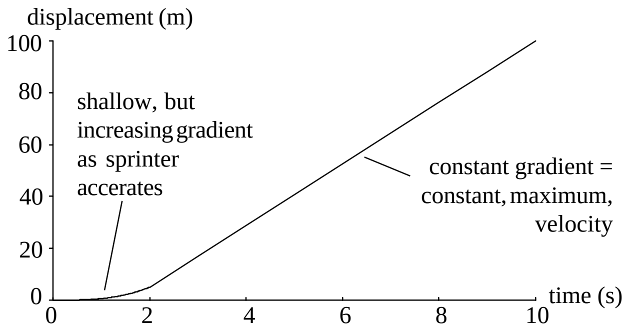
**Displacement-time graphs**

Displacement, plotted on the vertical axis, represents the straight line distance away from a start point. Time, plotted on the horizontal axis, is the time taken after the start.

- Since velocity = **displacement/time**, the gradient of a displacement-time graph also represents **velocity**. The steeper the gradient the larger the velocity.
- A straight line with a constant gradient will represent an object travelling with constant velocity.
- A curved line with a gradient that changes will represent an object travelling with a varying velocity.

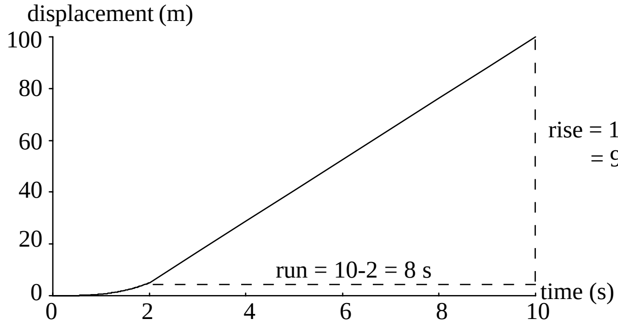



The graph below is a displacement-time for a 100 metre sprinter. The sprinter is slower at the beginning as it takes some time to reach full speed. This is shown by the shallow gradient during the first two seconds of the race, at the start of the graph. As the race progresses the sprinter reaches top speed and is able to maintain this maximum velocity for the rest of the race. This is shown by the gradient of the graph being constant after the first two seconds.




The size of the sprinter's maximum velocity can be obtained from the graph by calculating the gradient of the second section of the graph, beyond the two second point.

The gradient is best calculated by drawing a right angled triangle as shown in the diagram below. The height or 'rise' and length or 'run' of the triangle are then easily read from the graph and used to calculate the velocity.




$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{95}{8} = 11.86\ \text{ms}^{-1}$$

***Calculating velocity from a displacement-time graph***

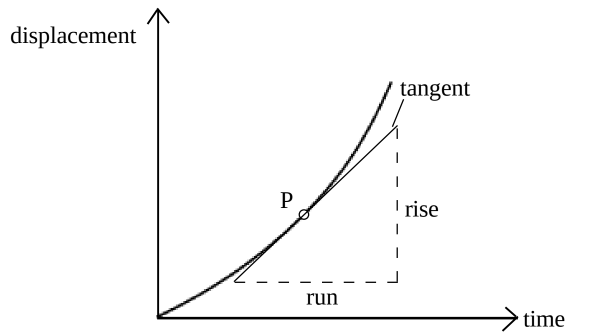
*The gradient of a displacement-time graph is equal to velocity.*

$$\textit{Velocity} = \textit{gradient} = \frac{\textit{rise}}{\textit{run}}$$

**Calculating instantaneous velocities from displacement-time graphs**

Calculating the gradient of a graph that does not have a convenient straight line portion requires a tangent to be drawn to the curve.

Consider the displacement-time graph below, which shows a constantly changing gradient indicating that the velocity of the moving object is constantly changing.




The instantaneous velocity of the moving object at point P will be given by the gradient of the curve at this point.

Calculating the gradient of the curve at this point is done by drawing a tangent to the curve. The tangent is the straight line that **just** touches the curve of the graph and has the same gradient as the graph **at this point**. The gradient of the tangent can then be calculated in exactly the same way as described previously, by forming a large right angled triangle and reading the 'rise' and 'run' of the triangle.

***Exam Hint:*** *It is a good idea to make the sides of your gradient triangle as long as possible. The reason for this is that a small mistake in a large number is not significant but a small mistake in a small number could easily be. Make sure you draw as long a tangent as you can – in order to make your gradient calculation as accurate as possible.*

*Look along your tangent, by holding you graph paper up to your eye. You can see how good it is and whether or not it just touches the curve at one point.*

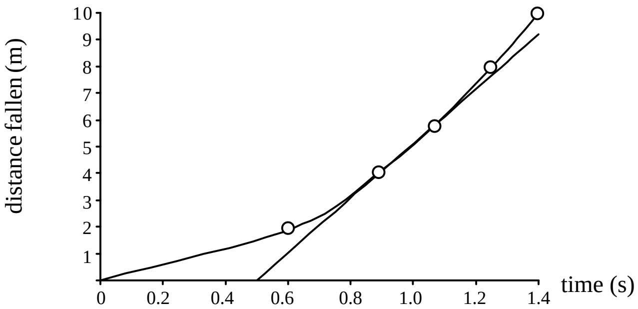
**Calculating instantaneous velocities from displacement-time graphs** - The instantaneous velocity can be calculated from a curved displacement-time graph by drawing a tangent to the curve at the place where the velocity is required. The gradient of the tangent to the curve will be equal to the instantaneous velocity at that point.

**Typical Exam Question**
**The table of results below were taken for an object being dropped and falling under gravity.**

| | | | | | | |
|---|---|---|---|---|---|---|
| **Distance fallen (m)** | **0.0** | **2.0** | **4.0** | **6.0** | **8.0** | **10.0** |
| **Time taken (s)** | **0.00** | **0.63** | **0.99** | **1.08** | **1.25** | **1.40** |

**(i) Plot a graph of distance fallen (on the vertical axis) against time taken (on the horizontal axis). [4]**
**(ii) Explain why your graph is not a straight line [2]**
**(iii) Calculate the velocity of the object after 1.00 second. [3]**

*Answer*
*(i) Graph paper would be supplied with a question like this one. Choose an axis scale that allows the plotted points to fill as much as the graph paper as possible. A mark may be deducted if your points don't fill more than half of the graph paper.*



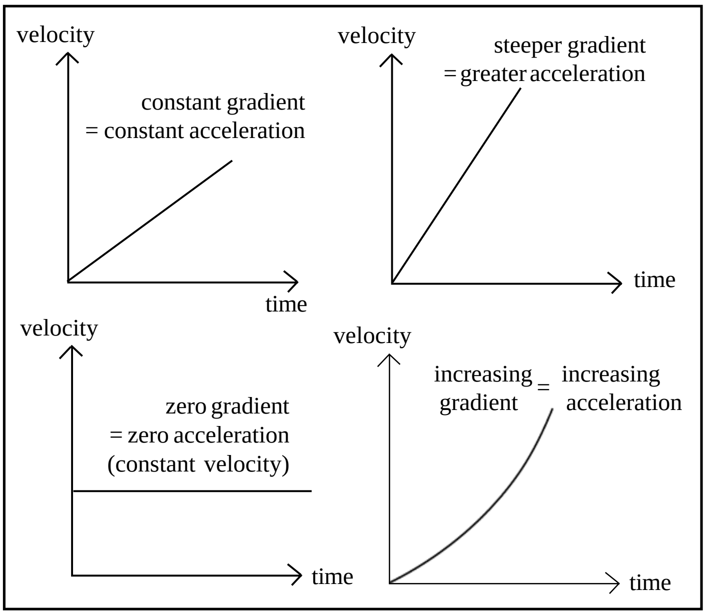

*(ii) The increasing gradient of the graph shows an increasing velocity, in other words acceleration.* ✓
*The acceleration is caused by the gravitational force acting on the object.* ✓

*(iii) The graph shown as the answer to part (i) has a tangent drawn at a time of 1.00 second.*
*Velocity = gradient of tangent* ✓

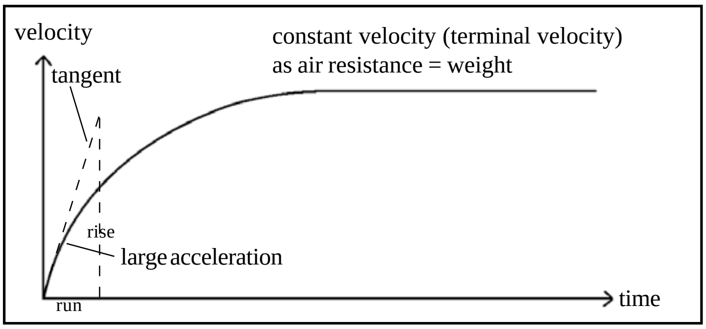
$$= \frac{rise}{run} = \frac{9.2 - 0}{1.4 - 0.5} \checkmark = 9.5 \text{ m/s} \checkmark$$

**Velocity-time graphs**
Velocity, plotted on the vertical axis, represents the velocity moving away from the start point. The time taken, plotted on the horizontal axis, represents the time taken since the start.

- Since acceleration = $\frac{\text{change in velocity}}{\text{time}}$
  the gradient of a velocity-time graph also represents acceleration.
- The steeper the gradient the larger the acceleration
- A straight line with a constant gradient will represent an object travelling with constant acceleration.
- A curved line with a gradient that changes will represent an object travelling with a varying acceleration.




The graph below represents the velocity-time graph for a freefalling skydiver.




The gradient is initially large as the skydiver is accelerating with the acceleration due to gravity.

The gradient of the graph gradually decreases showing the acceleration of the skydiver to be decreasing as the air resistance on the skydiver increases. Eventually the air resistance on the skydiver is equal to his weight; there is no resultant force so there is no acceleration. The skydiver falls at constant velocity, shown by the horizontal line on the graph; zero gradient implies zero acceleration and constant velocity.

The size of the initial acceleration of the skydiver can be determined by calculating the initial gradient of the graph. This is done in exactly the same way as for any other graph, by taking a rise and run from the graph, as we looked at with displacement – time graphs, using a right angled triangle drawn on the graph.

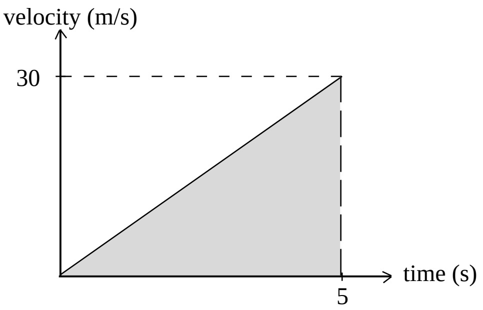
***Calculating acceleration from a velocity-time graph***
*The gradient of a velocity-time graph is equal to acceleration. The height or 'rise' and length or 'run' of a part of the graph is measured.*

$$Acceleration = \frac{\text{change in velocity}}{\text{time}} = gradient = \frac{rise}{run}$$

The total displacement during a journey can also be calculated from a velocity-time graph. The area beneath the line on a velocity-time graph gives the total displacement.

The graph below is a velocity-time graph for an accelerating car. The graph is a straight line showing that the car has constant acceleration. The displacement of the accelerating car after this 5 second journey can be determined by calculating the area beneath the graph.

The line of the graph forms a triangle with the horizontal axis so the area of the triangle can be calculated.




Total displacement = area beneath graph = area of triangle shape
= ½ × base × height = ½ × 5 × 30 = 75m

***Calculating the total displacement from a velocity-time graph***
*The total displacement is equal to the area beneath the line on a velocity-time graph for the time considered.*

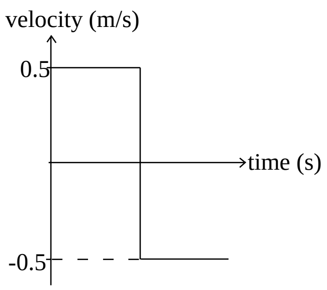
***Exam Hint:*** *The majority of velocity-time graphs that will be used in an exam will consist of sections of constant acceleration or constant velocity. This means that the graph can be split into a series of triangles and rectangles when calculating the area beneath the graph.*

**Velocity as a vector**

Velocity is a vector quantity. This means that velocities are described by two things; the size or magnitude of the velocity and the direction of the velocity.

- The size of the velocity is simply described by a number with a unit in the usual way, e.g a cyclist moving at 5 $ms^{-1}$.
- The direction of the velocity in journeys that can only go back and forth in a straight line is described by adding a sign to the size of the velocity.
- A positive sign would mean travelling in one direction and a negative sign would mean travelling along the same line but in the opposite direction. Therefore, a swimmer who is swimming lengths of the pool, there and back, would have a positive velocity while swimming to the far end of the pool but a negative velocity while swimming back to the start.
- Velocity-time graphs can also show negative velocities by having negative values plotted on the vertical axis.

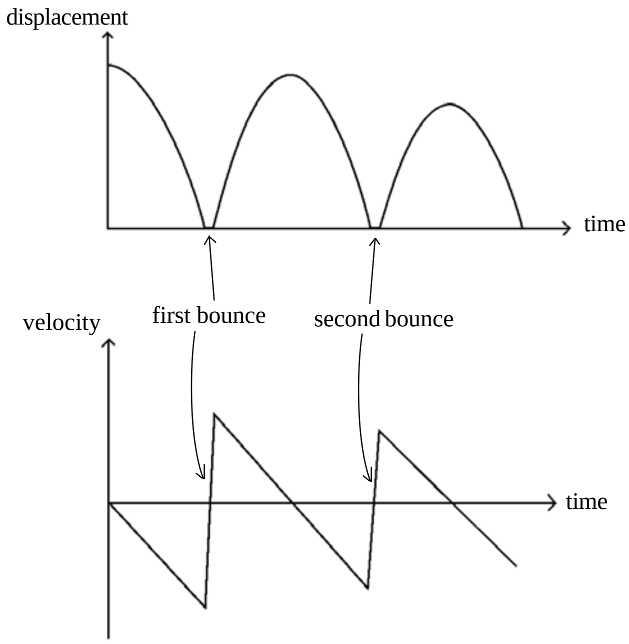
The swimmer, swimming at a constant velocity of 0.50 $ms^{-1}$, would have a velocity-time graph as shown below. The velocity switches from positive to negative as the swimmer turns around and starts to swim in the opposite direction along the pool.




***Velocity is a vector***
*Velocity is a vector. A vector is a measured quantity that is described by a magnitude, (or size), and a direction.*
*For motion along a straight line this means that moving in one direction along the line will be a positive velocity and moving in the opposite direction will be called a negative velocity.*

**Displacement-time graph and velocity-time graph for a bouncing ball**

The displacement-time and velocity-time graphs for a bouncing ball are specifically mentioned in several A-level specifications. The two graphs below are for a ball that is initially dropped from someone's hand and allowed to bounce on the floor.



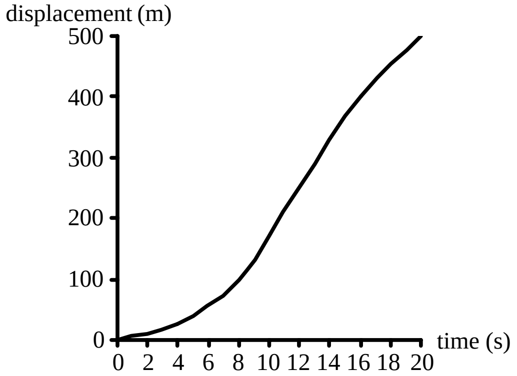

***Displacement-time graph***
- Zero displacement is defined as the floor.
- The gradient of the displacement-time graph is velocity. The gradient of the graph is negative and becomes increasingly large as the ball falls and speeds up.
- When the ball hits the ground, it bounces back up and the gradient becomes positive.
- The gradient then decreases until the ball is at the top of its path.
- The ball then drops downwards once more.

***Velocity-time graph***
- The ball is dropped from rest and so the initial velocity is zero.
- Velocity downwards has been given a negative sign and so the velocity then becomes a bigger negative number as the ball accelerates downwards.
- The gradient of the graph is acceleration and this is constant at –9.81 $ms^{-2}$ as this is acceleration due to gravity.
- When the ball bounces it rapidly comes to a stop before bouncing back, upwards, with a positive velocity.
- The ball will then slow down until, at the top of its path, it will instantaneously have zero velocity before heading back towards the ground.

**Exam Workshop**

**This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.**

**The graph below represents the displacement of a drag racing car along a straight track.**




**(a) (i) Calculate the instantaneous velocity of the car 12 seconds after the start. [2]**

$$velocity = \frac{displacement}{time} = \frac{250}{12} = 21\ ms^{-1}$$

The student has simply substituted values of displacement and time from the point at 12 seconds on the graph. No attempt has been made to determine the gradient of this straight line portion of the graph.

**(ii) Calculate the velocity of the car 6.0 seconds after the start. [3]**

$$velocity = \frac{displacement}{time} = \frac{50}{6} = 8.3\ ms^{-1}$$

Again, the values from the point at 6 seconds have been substituted. The graph is a curve at this point and a tangent should be drawn on the graph in order to calculate an instantaneous gradient.

**(b) On the axes below sketch a velocity-time graph for the car over the same period of time. [4]**


Even though the question says 'sketch' values should be placed on the vertical axis as we have just calculated 2 points from the first part of the question.

**(c) Without any calculation state what the area beneath your velocity-time graph represents and what the value should be. [2]**

*Area beneath the graph represents length of race and it should be 500m*

The candidate would be awarded both marks for this part of the question but more detail could have been given for the first part of the answer by mentioning the total displacement of the car from the start position.

**Examiner's Answer**

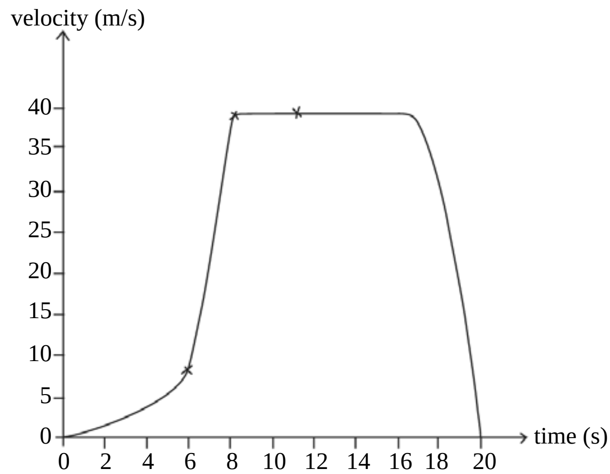
*(a) (i)* $velocity = gradient\ of\ graph = \frac{rise}{run} = \frac{400-100}{16-8} = 37.5\ ms^{-1}$ ✓✓

*(ii) Instantaneous velocity = gradient of tangent*

$= \frac{rise}{run} = \frac{115-0}{10-2} = 14.4\ ms^{-1}$ ✓✓

*Please note that actual numbers for rise and run will vary depending on the size of the line used to calculate the gradient but the final answers should all be very similar.*

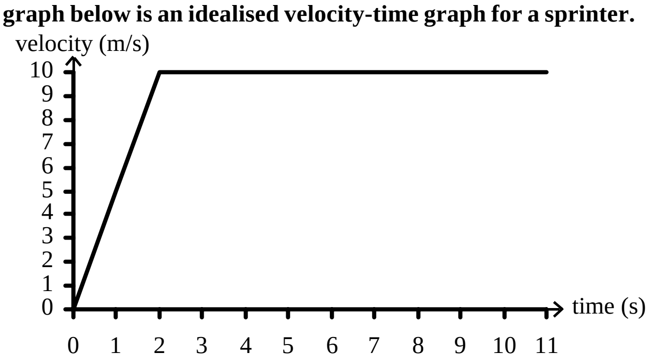
*(b)*




*(c) The area beneath the graph represents the total displacement of the car, which is the distance the car has travelled along the straight track.* ✓ *The area beneath the graph should be the total displacement given on the displacement-time graph = 500m.* ✓

**Typical Exam Question**

**The graph below is an idealised velocity-time graph for a sprinter.**




**(a) What is the initial acceleration of the sprinter? [3]**

**(b) Over what distance did the sprinter race? [3]**

**(c) What was the *average* velocity of the sprinter for the entire race? [2]**

*Answer*

*(a)* $acceleration = \frac{change\ in\ velocity}{time} = gradient = \frac{rise}{run} = \frac{10}{2} = 5\ ms^{-2}$ ✓✓✓

*(b) The displacement of the sprinter will give the length of the race. The area beneath the graph gives the displacement.* ✓ *The graph can be split up into a triangle for the first 2 seconds and a rectangle for the final 9 seconds. The total displacement will be given by the sum of the two areas.*

*Displacement = total area beneath the graph* ✓

$= (½ \times 2 \times 10) + (9 \times 10) = 100\ m$ ✓

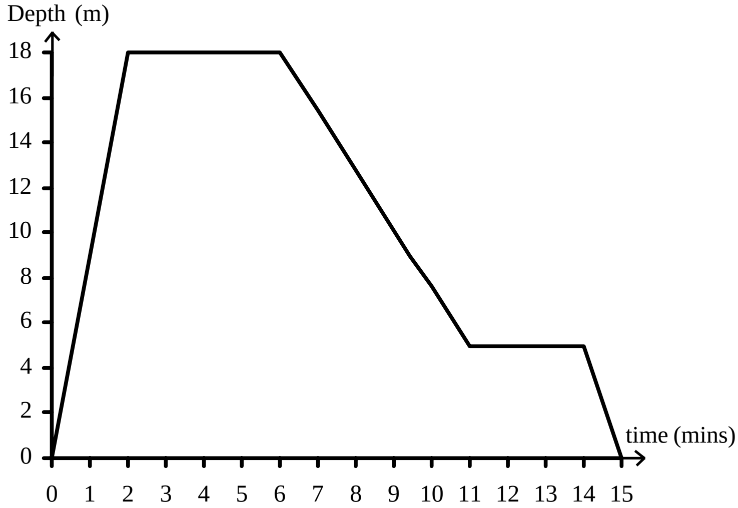
*(c)* $average\ velocity = \frac{total\ displacement}{total\ time\ taken} = \frac{100}{11} = 9.1 ms^{-1}$ ✓✓

## Qualitative (Concept) Test

1. What does the gradient of a displacement-time graph represent?
2. How would the gradient of a curved graph be calculated?
3. What does the gradient of a velocity-time graph represent?
4. What does the area beneath a velocity-time graph represent?
5. What is a vector and how is the vector nature of velocity in a straight line shown?
6. Sketch the displacement-time and velocity-time graph of a bouncing ball and label the important features of both.

## Quantitative (Calculation) Test

1. The graph below represents the depth of a scuba diver during a 15 minute dive.




(a) During which period of the dive was the diver ascending the quickest? [1]
(b) How long did the diver stay at the bottom of the sea, a depth of 18m? [1]
(c) What was the vertical velocity of the diver during his descent? [3]

2. The table of results below were taken for a cyclist travelling along a straight road.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Velocity ($ms^{-1}$) | 0 | 5 | 10 | 15 | 15 | 12 | 9 | 6 | 3 | 0 |
| Time taken (s) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |

(a) Draw a graph of velocity on the vertical axis against time on the horizontal axis for the journey. [4]
(b) What is the initial acceleration of the cyclist? [2]
(c) Calculate the deceleration of the cyclist in the final 50 seconds of the journey. [3]
(d) Calculate the total distance that the cyclist travelled along the straight road. [3]
(e) Calculate the average velocity of the cyclist for the entire journey. [2]

## Quantitative Test Answers

*1. (a) In the final minute of the dive.*
*(b) 4 minutes*
*(c)* $velocity = gradient = \frac{rise}{run} = \frac{18}{2 \times 60} = 0.15\ ms^{-1}$

*2. (b)* $acceleration = \frac{change\ in\ velocity}{time\ taken}$
$= \frac{(15-0)}{(30-0)} = 0.50\ ms^{-2}$

*(c)* $acceleration = \frac{change\ in\ velocity}{time\ taken} = gradient = \frac{rise}{run}$
$= \frac{(0-15)}{(90-40)} = -0.30\ ms^{-2}$
*deceleration =* $0.30\ ms^{-2}$

*(d) Total distance = area beneath graph*
$= (½ \times 30 \times 15) + (10 \times 15) + (½ \times 50 \times 15) = 750m$

*(e)* $average\ velocity = \frac{total\ distance}{time} = \frac{750}{90} = 8.3\ ms^{-1}$

***Acknowledgements:*** *This Physics Factsheet was researched and written by* ***Jason Slack.***
*The Curriculum Press,Unit 305B, The Big Peg,120 Vyse Street, Birmingham, B18 6NF.*

***ISSN 1351-5136***