Physics Factsheet

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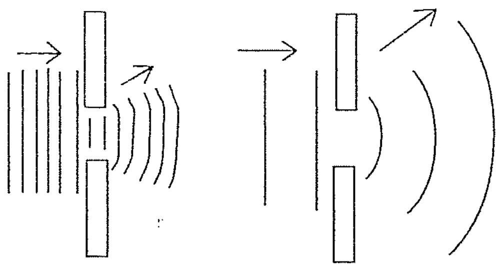
Diffraction Effects

Diffraction seems an unusual effect to most of us. We tend to think of waves or rays travelling in straight lines. They may reflect off a surface or change direction when entering a different medium, but we expect them to travel in a straight line otherwise.

The curving of waves within a medium caused by diffraction goes against our usual assumptions, and can be quite a problem at times. However we can also make use of this wave property.



Diffraction causes waves to bend around edges, or spread out through gaps. We often say that diffraction is a maximum when the wavelength is similar to the width of the aperture. If the aperture becomes smaller than the wavelength, diffraction will, in fact, increase. However the total wave energy passing through the gap decreases, making the diffraction effect less noticeable. So perhaps it is reasonable to think in terms of wavelength matching aperture size. It certainly makes calculations easier.

Strong diffraction occurs when the wavelength is equal to the width of the aperture.

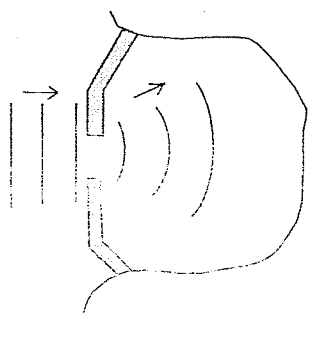
What we are going to look at in this Factsheet is diffraction, and its effects in a variety of wave motions (noting similarities and differences, uses and difficulties). We will also consider some basic mathematical relationships involving diffraction.

Answers to the questions posed will be given at the end of the Factsheet. It will prove useful to attempt each question as it occurs, rather than all together at the end.

A more in-depth look at diffraction with light, including more complex maths, will feature in a follow-up Factsheet.

Water waves

We build harbour walls with narrow entrances to shelter boats from large waves. But diffraction causes the waves to spread out within the harbour, limiting the value of the harbour walls:



Problem 1
(a) A wider opening would reduce diffraction within the harbour, but what are the problems with this?
(b) A narrower opening would increase diffraction. Is this necessarily counter-productive?

Another interesting diffraction effect can be seen with boats. A small boat often has a strong wave disturbance directly behind it, where a larger, wider boat does not.

Problem 2
Using ideas of diffraction and interference, can you explain this effect?

Sound waves

Sound travels at about 330 ms^{-1} through air (depending on temperature). This means that the wavelengths of audible sound tend to match aperture or obstacle size around us in nature (and in buildings). So diffraction effects are very large with sound waves.

Note that diffraction occurs both for transverse and longitudinal oscillations, and also for both mechanical and electromagnetic waves.

Problem 3
If we can hear sounds between 50 Hz and 20 000 Hz, find the range of wavelengths (in air) to which these frequencies correspond. (Notice how this range spans the size of many things around us, leading to diffraction.)

Owl hoots are at a relatively low frequency compared to songbirds. This longer wavelength leads to greater diffraction effects, bending the sound around trees and hillsides. The owl can communicate its presence over considerable distances. However the diffraction makes it very difficult for any prey to successfully locate the exact position of the owl.

Foghorns operate at low frequencies (long wavelengths). Diffraction means ships can hear the sound around (or over) islands and headlands that may be between the ships and the foghorn. However, again, locating the direction to the sound source is difficult.

Problem 4
Suppose a sound system is playing in the house. You are in the back garden with the door open. When you are in line of sight to the speakers, you hear more treble; when you are off to the side you hear more bass. Explain this.

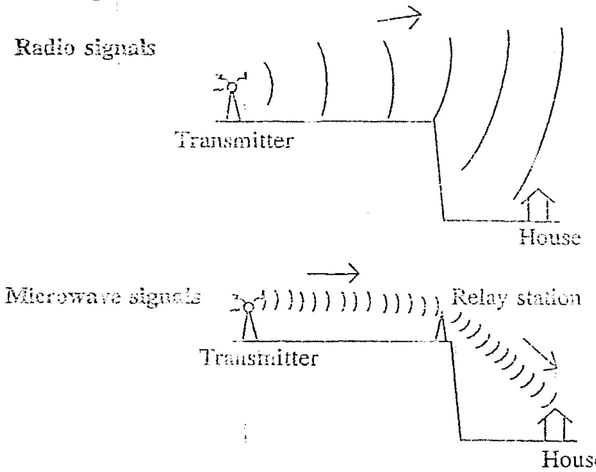
In a concert hall, it is important that the sound from the speakers fans out across the hall. But it is also important to limit the sound intensity reaching the ceiling, as the reflections will increase echoing (reverberation) within the hall.

Problem 5:
What shape would you choose for the loudspeaker apertures within the concert hall? (Should they be tall and thin, or short and wide?)

Radio Waves and Microwaves

Diffraction is useful for radio transmission, especially for long and medium wave signals. These longer wavelengths will diffract over hills and around buildings, making the signal accessible in 'awkward' areas.

However FM radio and television make use of shorter wavelengths for their carrier frequencies, extending into the microwave region. Shorter wavelengths mean less diffraction. Relay stations are needed to maintain 'line of sight' transmission.



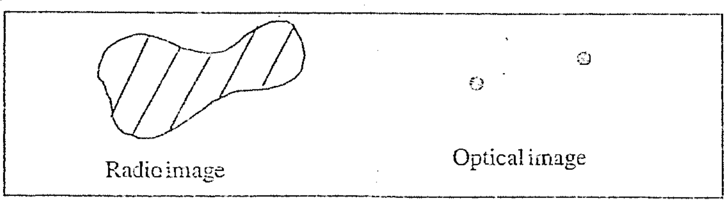
Radio telescopes are used to study radio transmissions from space. We use the term **resolution** to describe the ability of a device to separate the images of two sources which have only a small angular separation.

The resolution of a telescope (optical or radio) depends on the ratio of the wavelength to the aperture. To minimise diffraction (and improve resolution), the aperture should be as large as possible, compared to the wavelength being observed.

Limit of resolution $\propto \lambda/a$ *where a is the diameter of the aperture.*

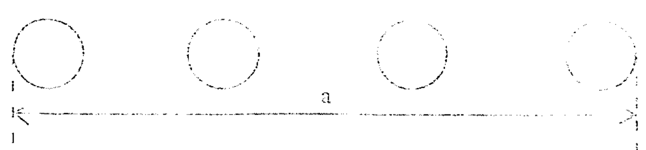
> **Problem 6**
> Suppose an optical telescope of lens diameter 1m is used to observe light of wavelength 5×10^{-7} m. What diameter must the reflector of a radio telescope be in order to produce the same resolution when observing radio waves of wavelength 10 cm?

Obviously we cannot build radio dishes with diameters measured in kilometres (see solution to Problem 6). Resolution is much less with radio telescopes than optical telescopes. Radio sources appear as blurs, while the same sources can be observed as pinpoints optically.



It might then seem pointless studying radio emissions, when the images from optical observations are so much sharper. However the radio data provides different information, which might be useful in its own right. Sometimes optical observations are made to positively identify the source of the radio waves.

One way in which resolution can be improved with radio telescopes is by linking the signals received from a line of small dishes:



The effective aperture, a, is very much increased using this setup, improving the resolution. However the total energy collected (the 'brightness' of the signal received) depends on the area of the dishes. So this system increases resolution significantly, but does not do much for the intensity of the signal received.

> **Problem 7**
> A line of 9 dishes, each of diameter 1.0m with a 50.0m gap between adjoining dishes, is used to detect radio emissions from a source in space. Compare the energy collected by these dishes with that which would be collected by a dish with a real diameter equal to the effective diameter of this system.

One place you may have seen microwave diffraction in the laboratory is to model X-ray or electron diffraction in crystallography. X-rays and electrons can have wavelengths similar to the spacing between atomic planes. We can simulate the diffraction effects observed with model atomic lattices of plane spacing similar to the wavelength of the microwaves used (about 3 cm).

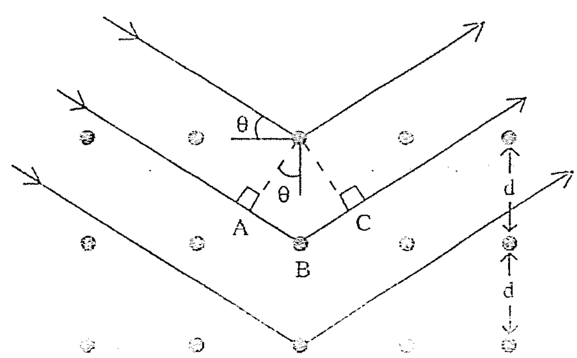
X-rays and Electrons

It may seem strange to group these topics, but both X-rays and electrons are used to study atomic structure.

X-rays are, of course, high energy electromagnetic radiation. They are produced by bombarding a metal target with a beam of accelerated electrons. The X-rays produced have wavelengths spanning those required to undergo diffraction by crystal planes.

We usually think of electrons as particles, but the wave-particle duality theory tells us that they have a wavelength associated with them, and electrons of the right energy will display diffraction effects when directed into crystals.

The diffraction theory is similar for both:



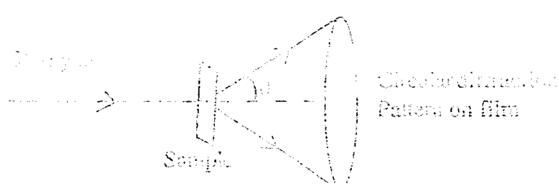
The receiver will measure an interference maximum when the extra distance travelled by the second ray is a whole number of wavelengths:

$n\lambda = 2d \sin \theta$, where n=1, 2, 3, etc.

(The "2" in this equation comes from the path difference between two adjacent rays being AB + BC in the diagram.)

If we know the wavelength and measure the angle, we can work out the plane spacing for the crystal. (This is usually of the order of 10^{-10} m.)

If we select one specific known X-ray wavelength (using filters), then the crystal would have to be oriented at exactly the right angle to give a diffraction pattern. But if we use a powder (or an equivalent), where the crystal grains are randomly at all possible orientations, we will always get a diffraction pattern.



Problem 8
If a beam of X-rays of wavelength 2.1×10^{-10} m is directed at a thin metal sheet (composed of a large number of randomly oriented metal grains), a diffraction pattern is produced. It is found that rays deflected through 36 degrees produce the first interference maximum. Find the spacing between the atomic planes in the metal.

An alternative to these transmission patterns, where the X-ray beam penetrates the sample, is the back reflection method. Diffraction patterns are obtained from X-rays reflected back from the surface of a sample.

With electrons, we select the wavelength by choosing the voltage through which we accelerate them.

$E = eV$ *(usually quoted in electron-volts)*

$p = mv = h/\lambda$ *(de Broglie's equation)*

h is Planck's constant
m and e are the rest mass and charge of an electron.

De Broglie's equation links wave and particle properties of matter.

By combining these equations (and remembering that $E = \frac{1}{2}mv^2$), we reach our final relationship:

$$\lambda = h / \sqrt{(2meV)}$$

Problem 9
(a) If we accelerate an electron through 200V, what energy do we give it (stated in joules)?
(b) What wavelength would this energy translate to for an electron?
(c) Would this be suitable for studying crystal structure using electron diffraction?

Problem 10
What voltage would produce electrons with a wavelength of exactly 3.0×10^{-10} m?

Electron diffraction can also be used to study the nucleus. Here we need wavelengths of about 10^{-15} m for diffraction patterns. If you use the above equations, you will find that the speeds required of the electrons are greater than the speed of light.

However it is possible to produce these wavelengths. As the electrons approach the speed of light, relativistic calculations replace the above equations with the simple expression:

$$E = eV = hc/\lambda$$

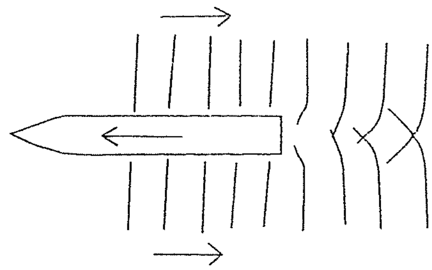
A quick calculation will show you that if you can accelerate the electrons through several million volts, you can achieve very short wavelengths.

Visible Light
A separate Factsheet will deal with diffraction effects with visible light. Factsheet 81 will include:

(a) Single slit diffraction (through gaps and holes)
(b) Multiple slits
(c) Transmission and reflection gratings
(d) Crossed gratings
(e) Resolution with optical devices
(f) Mathematical calculations with the above topics
(g) Difficulties caused by diffraction
(h) Uses of diffraction e.g. spectroscopy

Answers
Problem 1 Solution
(a) More wave energy would enter the harbour. Reflections from inner harbour walls could still spread the wave energy throughout the harbour. So increasing the harbour opening is not a sensible option.
(b) Although diffraction would increase, the total wave energy entering the harbour would decrease, reducing the danger to the boats sheltering in the harbour. However a narrower harbour entrance would make navigation into the harbour more difficult in poor weather conditions.

Problem 2 Solution
With a narrower boat, waves diffracting around the back corners can meet behind the boat, leading to a strong interference effect.



(My apologies to readers if there is a nautical term for 'back corners'.)

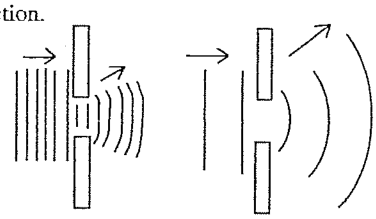
Problem 3 Solution:

$$\lambda = \frac{v}{f} = \frac{330}{50} = 6.6\text{m}$$

$$\lambda = \frac{v}{f} = \frac{330}{20\,000} = 0.017\text{m} = 1.7\text{cm}$$

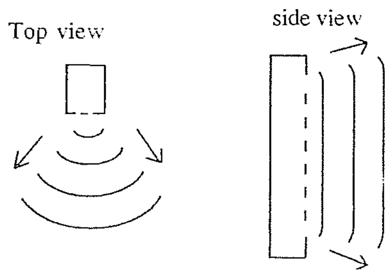
This range of wavelengths for audible sound makes diffraction very noticeable.

Problem 4 Solution:
Through the same aperture, longer wavelengths (bass notes) experience greater diffraction.



A greater proportion of the energy from the bass notes will be diffracted through greater angles. The 'line of sight' sound will have lost more energy from the bass, sounding more treble. Off to the side, the music will sound more bass (deeper).

Problem 5 Solution:
The speakers should be tall and thin.



Diffraction from the narrow horizontal aperture fans the sound out across the hall.

The large vertical aperture reduces upward diffraction. This minimises sound energy reflecting from the ceiling.

Several other measures are taken to optimise the acoustics within the concert hall, sometimes by manipulating other properties of waves. But

Problem 6 solution

$(\lambda/a)_{optical} = (\lambda/a)_{radio}$

$$\frac{5 \times 10^{-7}}{1} = \frac{0.1}{a}$$

$$a = \frac{0.1}{5 \times 10^{-7}} = 200\,000 \text{ m} = 200 \text{ km.}$$

So the radio dish required to produce the same resolution would have a diameter of 200 km.

Problem 7 solution

Diameter of big dish = (9 × 1.0) + (8 × 50.0) = 409 m

Area = $\pi \times r^2 = \pi \times 205 \times 205$ = 130 000 sq.m.

For small dishes:

Area = $9 \times \pi \times 0.5 \times 0.5$ = 7.1 sq.m.

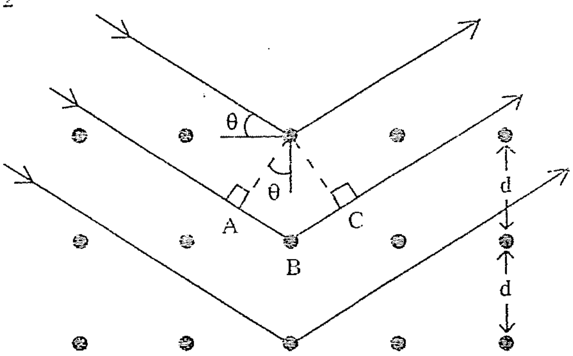
The ratio of energies collected will be the same as the ratio of areas = 130 000 : 7.1 = 18 000 : 1

Problem 8 Solution:

$n\lambda = 2d \sin\theta$

$n=1, \ \lambda=2.1 \times 10^{-10}$ m

$\theta = \frac{36}{2} = 18$ deg (see diagram)



$$d = \frac{\lambda}{(2 \sin\theta)} = 3.4 \times 10^{-10} \text{ m.}$$

Problem 9 Solution:

(a) $E = 200 \text{ eV} = 200 \times 1.6 \times 10^{-19} \text{ J} = 3.2 \times 10^{-17}$ J

(b) $$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 200)}} = 8.6 \times 10^{-11} \text{ m.}$$

(c) This seems reasonable. The spacing of planes in crystals is typically 3 × 10^{-10} m.

Problem 10 Solution:

Rearranging the given equation:

$$V = \frac{h^2}{(2me\lambda^2)} = 16.6\text{V}$$

Typical exam question

We will conclude with a (fairly simple) typical exam question for you to attempt:

(a) Define diffraction (in words).

(b) What relationship between wavelength and aperture size leads to maximum diffraction?

(c) Wave motion of wavelength, λ, gives a diffraction pattern when it goes through an aperture of width, a. What happens to this pattern if we:
(i) Double the wavelength (only)
(ii) Double the aperture width (only)
(iii) Double both wavelength and aperture width?

(d) For multiple slit diffraction, the fringe spacing, y, is given by the equation:
$y = \lambda D / d$, where D is the distance from slits to screen, and d is the slit spacing.

(i) Microwaves of wavelength 3.0 cm travel through a double slit of spacing 6.5 cm, and the detector is placed 0.8 m beyond the slits. Find the distance between adjacent maxima in the interference pattern.

(ii) The microwave source and detector are replaced by an infrared source and detector. What would you notice about the diffraction pattern?

Answers

(a) *Diffraction is the bending of a wavefront when it travels through an aperture or past an edge.*

(b) *Wavelength and aperture size should be approximately equal.*

(c) (i) *The pattern spreads out.*
(ii) *The pattern closes in.*
(iii) *The pattern is unaffected.*

(d) (i) $y = \frac{\lambda D}{d} = \frac{0.03 \times 0.8}{0.065} = 0.37$ *m*

(ii) *The wavelength is much shorter for infrared. The fringe spacing would therefore be much smaller. You probably would not be able to see a diffraction pattern.*

Acknowledgements:
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