Physics Factsheet

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Diffraction and Light

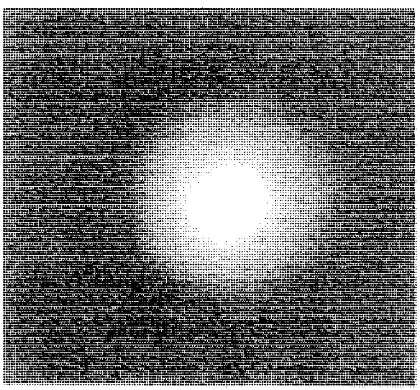
n Factsheet 78 we looked at diffraction effects across a wide range of wave notions – both electromagnetic and mechanical. This time we will focus on liffraction specifically with visible light.

Why? Because our eyes are more sensitive to this range of wavelengths han any other. In addition, diffraction causes limitations in the resolution of optical instruments, and can also be used in the study of atomic energy evels (through spectroscopy). Also, some diffraction effects can be seen by the naked eye, and demand explanation.

Atmospheric Diffraction

An example of a diffraction effect we see in nature is atmospheric diffraction. Light can bend around water droplets in thin clouds.

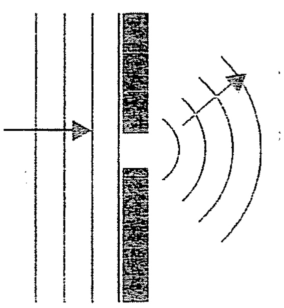
The black-and-white photograph shows a coloured diffraction ring which can appear when the Sun is low in the sky on a misty morning. Similar liffraction rings can be seen around the Moon.



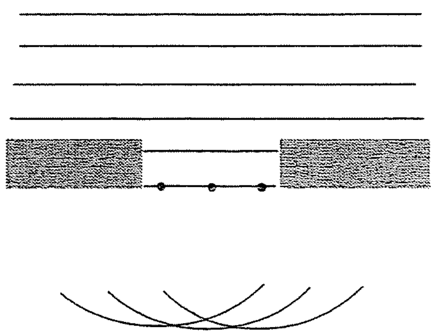
The "silver lining" we can see around dark clouds is also a diffraction effect.

Single Slit Diffraction

We know that light will spread out (diffract) on going through a gap, and hat the diffraction effect is greater for a longer wavelength or narrower gap.

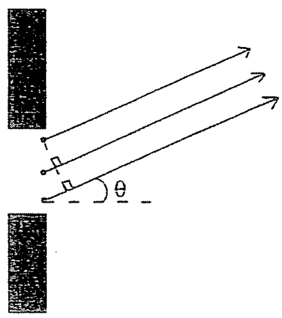


Huygen's Theory says that we can think of each point on a wavefront as a new source of waves. This gives us a nice visual idea of how diffraction occurs when we send light through a gap. The resultant wavefront curves at the ends.



This wavefront spreads out as it travels forwards. When it reaches a screen, we see a diffraction pattern displayed.

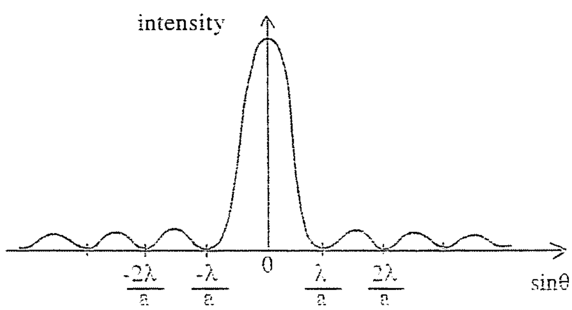
This is an interference effect, caused by light rays from various parts of the gap travelling different distances to reach a point on the screen. They may arrive in or out of phase, causing constructive or destructive interference.



The dark fringes (destructive interference) occur when:

$\sin\theta = n\lambda/a$, where n = 1,2,3,... and where a is the width of the gap

A graph of the intensity across the screen might look like this:

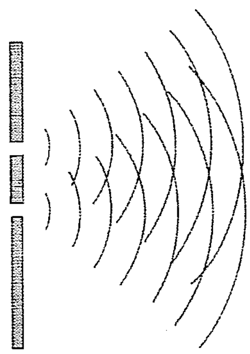


Problem 1:
(a) For light of wavelength 5.0×10^{-7}m incident on a gap of width 7.5×10^{-6}m, find the angles for the first two dark fringes.
(b) If the screen is 1.5m from the aperture, find the width of the central maximum.
(c) Find the width of the central (zeroth order) maximum if the width of the gap is doubled.

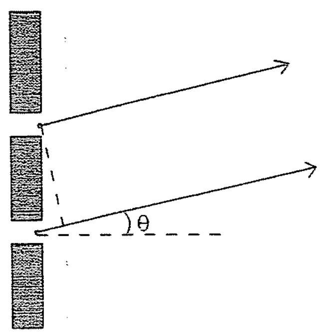
Notice that making the aperture wider causes the interference pattern to close in.

Two Slit Diffraction

If we send coherent wavefronts through two narrow gaps separated by a spacing of 'd' metres, an interference pattern is again formed. Waves spreading out (diffracting) from the two slits overlap to form a set of bright and dark equally spaced fringes on the screen.



Thinking of each slit as a point source, we can draw a ray diagram.

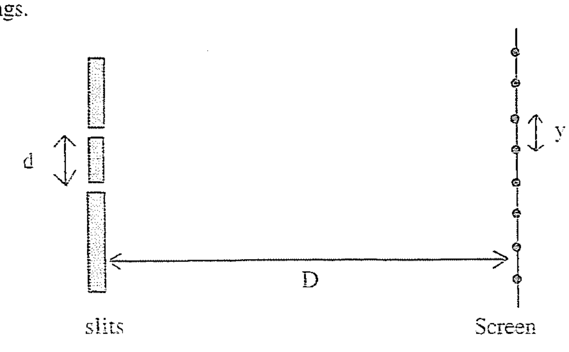


And the maths tells us that maxima will be seen when:

$\sin\theta = n\lambda / d$, where n = 0,1,2,3....

Notice that this expression gives the position of the maxima. Compare this to the single slit diffraction equation.

There is another useful expression for multi-slit diffraction and diffraction gratings.



If the screen is at a distance of D metres from the slits, and the fringe spacing observed on the screen is y metres, then we can show that:

$\sin\theta = \tan\theta$ (approximately) $= y/D$ (for small angles)

So we can write the fringe spacing, y, as:

$y = \lambda D / d$

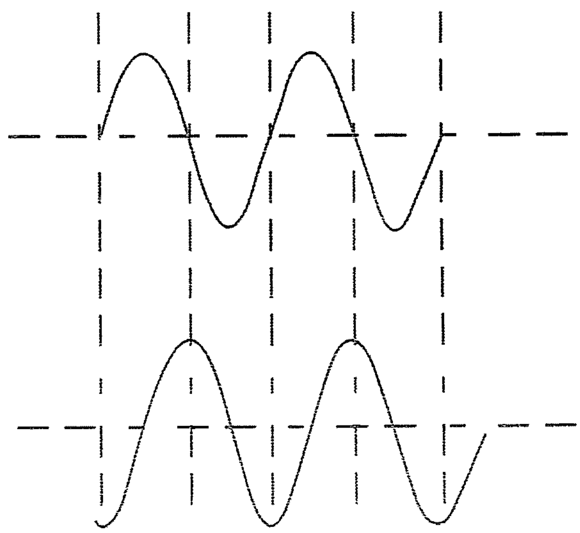
Problem 2:
With a plane wavefront of light of wavelength 6.5×10^{-7}m incident on a twin slit of spacing 5.2×10^{-5}m, find the fringe spacing on a screen at a distance of 2.5m from the slits.

Problem 3:
Consider the set-up in Problem 2 again. What would the effect be in each case of:
(a) doubling the wavelength?
(b) doubling the spacing between the slits?
(c) doubling both at the same time?

Exam Hint: *Be prepared for questions concerning the effect of changing wavelength, aperture size, slit spacing, monochromatic light to polychromatic light, etc.*

Phase and Coherence:

The **phase difference** between 2 waves travelling together depends on the fraction of a wavelength which one lags behind the other.



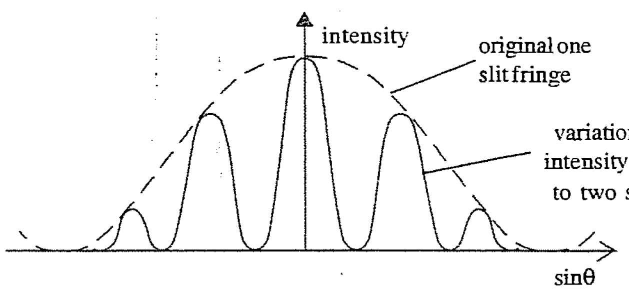
The phase difference here is one-quarter of a wavelength (or 90 degrees).

We say that two wave sources are **coherent** if there is a constant phase difference between them.
For our double slit diffraction experiment, as long as the waves reaching each slit are coherent, our interference theory still works. The only difference evident would be that the whole interference pattern would be slightly displaced one way or another along the screen (compared to the waves being completely in phase).

Multiple slit diffraction (or a diffraction grating) requires coherent wavefronts. The light must come from a single source.

Combination Effects

We looked at double slit theory on the assumption that we could treat each slit as a point source. However each slit has an aperture of width 'a'. The double slit pattern we have observed is actually superimposed on the relevant single slit pattern.

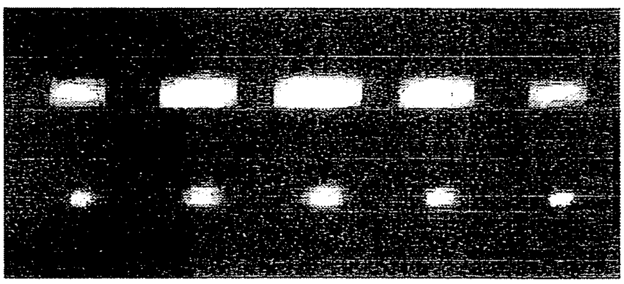


The double slit pattern will fade in and out as it fits within the envelope of the single slit pattern.

> **Problem 4:**
> For a double slit diffraction set-up, it is noticed that the fifth order maximum (n=5) disappears, as it is superimposed on the first order minimum of the single slit pattern. If the slit spacing, d, is 4.0×10^{-5}m, find the aperture width, a, for each of the slits.

Multiple Slits

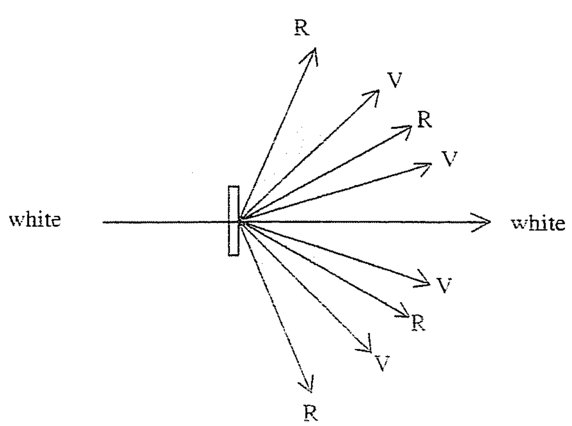
If we have more than two slits, but the values of slit aperture, a, and slit spacing, d, remain unchanged, then our mathematics remains the same. What we notice is that the bright fringes are **narrower** and more **intense**.



Monochromatic and Polychromatic Diffraction

So far we have assumed a single wavelength (monochromatic). If we use a white light source, we will get spectra, rather than bright bands, for our maxima.

As red light has the longest wavelength, by looking at our theory we can see that red will be on the outside of each spectrum formed, and violet on the inside. The central maximum will be white as no path difference is involved.



For larger angles, adjacent maxima for different colours will overlap each

> **Problem 5:**
> Suppose we take the wavelength of red and blue light to be 7×10^{-7}m and 4×10^{-7}m. For multiple slit diffraction, for what order, n, would the red maximum be outside the blue maximum for order, (n+1)?

Diffraction Gratings

A diffraction grating is a plate with many close, regularly spaced slits (or rulings). There may be thousands per millimetre. Nowadays gratings need no longer be physical scorings on a glass plate. They can be produced on photographic film and created from a holographic interference pattern.

The patterns from a grating are the same (in theory) as those from multiple slits. However the greatly increased number of slits lets much more light through. And increased interference sharpens the maxima observed.

> *A diffraction grating produces brighter and sharper maxima. Spectra produced from polychromatic sources are better defined using a grating. The theory is the same as for multiple slits, or even just two slits.*

Typical exam question

(a) A diffraction grating has 850 lines per mm. Find the slit spacing for this grating.

(b) Find the angle for the first maximum for a wavelength of 5.0×10^{-7}m.

(c) The diffraction pattern obtained using a source of this wavelength is displayed on a screen. State any changes you would observe in the position or appearance of this pattern if the grating was blacked out except for two adjacent slits.

(d) Describe the pattern observed if a white light source was used.

Answers

(a) 850 *lines/mm* $= 8.5 \times 10^5$ *lines / m.*

Slit spacing, $d = \dfrac{1}{8.5 \times 10^5} = 1.2 \times 10^{-6}$ *m.*

(b) $\sin\theta = \dfrac{n\lambda}{d} = \dfrac{1 \times 5.0 \times 10^{-7}}{1.2 \times 10^{-6}}$

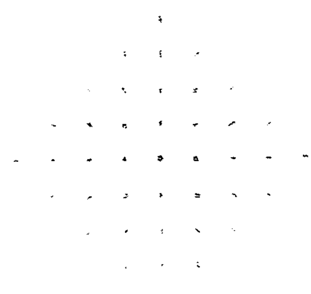
$\theta = 25$ *degrees.*

(c) The pattern would be less bright, as less light would reach the screen. The pattern would not be as well defined (sharp) with two slits as with a grating.

(d) You would see a central (zero order) white maximum. The other maxima would be spectra with the violet on the inside of each spectrum (closer to the straight-through direction.

Reflection gratings work just as well as **transmission** gratings. If you hold a CD up to the light you can see interference spectra resulting from the diffraction of white light incident on the grooves. This is often more interesting than the music on the CD.

With **crossed** gratings we can produce a two-dimensional interference pattern.

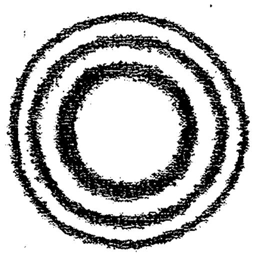


This pattern can also be obtained if we look at a bright point source

Resolution

Stars can be considered as point sources of light. But even the most powerful and optically perfect telescopes cannot distinguish two stars if the angle separating them is too small. This problem is caused by single slit diffraction at the aperture of the telescope.

A circular aperture produces a circular diffraction pattern with a central bright spot.



If two patterns are closely superimposed, it may be impossible to separate them. The two stars may appear as one blur. We have previously discussed this problem with radio astronomy.

Example:
Explain why astronomers prefer larger aperture telescopes, and may use blue or violet filters to observe the light from stars.

Answer:
Larger apertures and shorter wavelengths reduce the diffraction effects, increasing the resolving power. These changes may allow us to resolve the two stars. The equation for resolving two point sources says that their angular separation must be great enough so that:

$\sin\theta > 1.22\lambda/a$

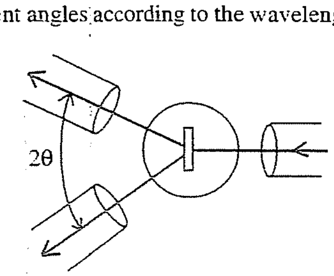
A smaller wavelength and a larger aperture lead to a smaller angle being resolved.

In astrophysics / cosmology you may be required to solve numerical problems on resolution.

> **Problem 6:**
> What angular separation would be required to just resolve 2 stars observed through a telescope of aperture 30 cm using a filter allowing through light of wavelength 4.5×10^{-7}m?

Spectrometer:

Factsheet 75 (Line Spectra) discusses how emission spectra are produced by viewing a discharge tube through a spectrometer. The diffraction grating in the spectrometer gives rise to a series of discrete coloured lines diffracted through different angles according to the wavelength of the light.



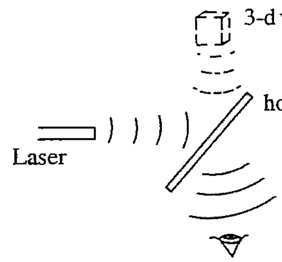
> **Problem 7:**
> For a certain line in the spectrum from a gas discharge lamp, the angular separation of the first order maxima (each side of the straight through direction) is 72 degrees. The diffraction grating has 1200 rulings per mm.
> (a) Find the wavelength of the light emitted.
> (b) What would the angle of diffraction be for the second order line for this wavelength of light?

Holography

Holography could easily be a topic on its own. We will just mention it for completeness in this study of optical diffraction.

A beam splitter allows a reference beam from a laser (coherent light at a single frequency) to interfere with a beam reflected from the object. Photographic film is used to store this interference pattern.

If the laser is then directed onto this interference pattern, diffraction occurs.

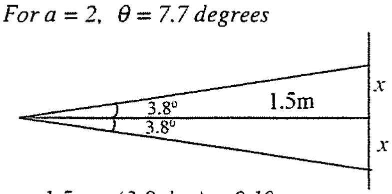


The viewer sees a virtual image of the object behind the film. Reflection holographs can also be used to recreate the image.

Solutions

Problem 1 solution:
(a) $\sin\theta = n\lambda/a$
For $a = 1$, $\theta = 3.8$ degrees
For $a = 2$, $\theta = 7.7$ degrees

(b)



$x = 1.5 \tan(3.8 \text{ deg}) = 0.10$m
The width of the central maximum is 0.20m.

(c) $n=1$
$\sin\theta = (5.0 \times 10^{-7}) / (1.5 \times 10^{-5}) = 1.9$ degrees.
The central maximum then works out to be 0.10m.

Problem 2 Solution: $y = \lambda D/d = 0.031$m

Problem 3 Solution:
(a) *This doubles the fringe spacing (spreads out the pattern).*
(b) *This halves the fringe spacing (closes in the pattern).*
(c) *The two changes should cancel each other.*

Problem 4 Solution: $\sin\theta = \lambda / a$ coincides with $\sin\theta = n\lambda / d$
$\lambda / a = n\lambda / d$
$a = d / n$, $a = 8.0 \times 10^{-6}$m.

Problem 5 Solution:
If the two maxima coincide with each other:
$\sin\theta = n\lambda(red)/ d = (n+1)\lambda(blue)/d$
$n\lambda(red) = (n+1)\lambda(blue)$
$7 \times 10^{-7}n = 4 \times 10^{-7}(n+1)$
$n = 4/3 = 1.33$
So the red maximum would be outside the next blue maximum for n=2.

Problem 6 Solution: $\sin\theta = 1.22\lambda / a = 1.22 \times 4.5 \times 10^{-7} / 0.30$
$\theta = 1.0 \times 10^{-4}$ degrees.

Problem 7 Solution:
(a) $\sin\theta = n\lambda/d$ where $n = 1$ and $\theta = 36$ degrees.
$d = 1 / 1200 = 8.3 \times 10^{-4}$mm $= 8.3 \times 10^{-7}$m.
$\lambda = d \sin\theta/n = 4.9 \times 10^{-7}$m.

(b) $\sin\theta = (2 \times 4.9 \times 10^{-7})/(8.3 \times 10^{-7}) > 1$. *There is no second order spectrum obtainable. You would have to use a coarser grating (increase d) to make θ less than 90 degrees.*

Acknowledgements:
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