Physics Factsheet

September 2001 Number 23

Circuit Electricity I

In this Factsheet we will:
- **briefly explain what current, voltage and resistance are;**
- **investigate how and why electrical circuits function;**
- **look at examples of questions on electrical circuits.**

Vital to an understanding of electrical circuits is a basic knowledge of what current, voltage and resistance are. A more full explanation can be found in Factsheet No. 07 - Electrical Current, Voltage and Resistance.

Introduction

An **electrical current** is a net movement or flow of charge in a given direction. In a metal, this charge is negative and is due to the movement of free electrons within the structure of the metal. You can imagine current as behaving in a similar way to cars in a nose-to-tail traffic queue or as molecules in an incompressible fluid. Cars at the rear of a traffic queue can only move forwards if the cars at the front do - otherwise they have no space to move in to. In a similar way you cannot depress the plunger of a water-filled, plastic syringe if the other end is blocked - the water molecules have nowhere to go. For this reason it is necessary to have a complete circuit loop, with no breaks, for a current to flow. We will examine circuit loops in more detail later on. This also means that current will be the same at any point around a series circuit - again more later on.

> ***Exam Hint:*** *- Remember conventional current (the direction we assume the charge travels in a current) is in the opposite sense to the direction of electron travel (the direction they actually travel).* ***Conventional current flows from positive to negative, electrons travel from negative to positive.*** *You should always mark conventional current on diagrams unless specifically asked to do otherwise.*

- A v**oltage** is a measure of how much potential energy a unit charge has at a point, specifically here in an electric circuit.
- A **potential difference** exists between two points if a charge has a differing value of potential energy at each of the points. A **voltage drop** means the charge has lost energy; an **emf** (electromotive force) means the charge has been supplied with energy.

The easiest type of p.d. to understand is that produced by a cell – which contains a surplus of electrons in the negative terminal and a lack in the positive terminal. Electrons in the negative side of the terminal are repelled by the like charge surrounding them and pushed out into the circuit to fill the gaps left by electrons drawn towards and into the positive terminal. Just as a mass falling to earth loses gravitational potential energy, electrons moving towards a positive terminal lose electrical potential energy, and it is this eletrical potential energy that is transferred into heat.

Resistance in a metal is due to the presence of fixed, positive ions which inhibit the flow of electrons through the metal. As electrons collide with the positive ions, the electrons give up energy and it is transferred to the structure of the metal, normally resulting in an increase in temperature, which causes heat to be transferred to the surroundings. Fundamentally this is how an electric heater and filament lamp function.

In brief, an electromotive force causes electrons to move around a complete circuit giving up energy as they progress. The current transfers energy from the cell to components in the circuit.

Kirchoff's Laws

As already mentioned, current in a wire is composed of electrons. When this flow comes to a junction, unless they leak out of the wire (which they don't) the number of electrons that flows in must equal the number which flows out. This is **Kirchoff's First Law.**

> **Kirchoff's First Law** states that the sum of currents flowing into a junction must equal the sum of currents flowing out of a junction. It can be written using the shorthand $\Sigma I = 0$ and it is fundamentally a statement of conservation of charge.

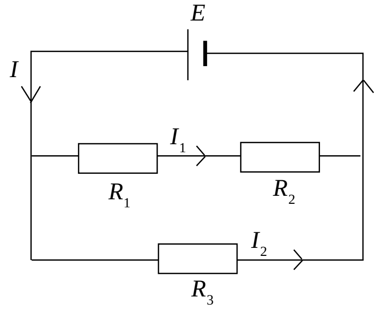
As you already know energy is always conserved.
- An emf is a measure of energy transformed into electrical energy per unit charge;
- voltage drop is a measure of the energy transformed from electrical energy to other forms per unit charge.

Bearing in mind conservation of energy it stands to reason that the total energy supplied to each electron will equal the amount of energy lost. In other works the sum of emfs must equal the sum of voltage drops.

> **Kirchoff's Second Law** states that the sum of the potential differences across components in any complete loop around the circuit must equal the sum of the electromotive forces supplying it. In other words $\Sigma E = IR$. This is fundamentally a statement of the conservation of energy.

> ***Remember*** *- choose a direction around the circuit. If the emf opposes you then it must be given a negative sign.*

For example:



From K1: $I = I_1 + I_2$

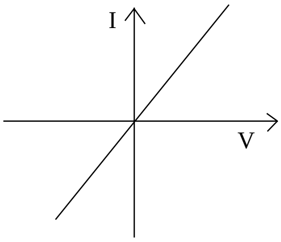
From K2: $E = V_1 + V_2$ and also from the other loop $E = V_3$.

Even if you do not meet Kirchoff's laws by name you will us them implicitly in circuit calculations, as you will see later in this Factsheet.

Different components respond differently to different voltages and currents.

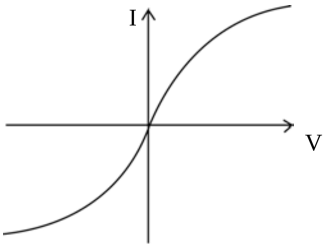
I-V characteristics

1. Current-carrying wire at a constant temperature.



For this component I ∝ V. When a component behaves in this way it is said to obey Ohm's law. **Ohm's law states that the current through an ohmic conductor is directly proportional to the voltage across it provided its temperature is constant**. The resistance is constant and is given by the inverse of the gradient (R = 1/m). If we double the voltage, the current will also double.

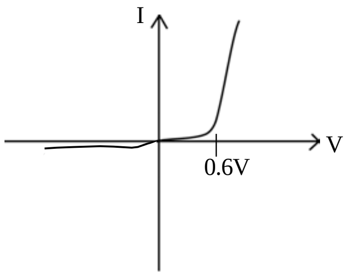
2. Filament lamp.



Here, resistance is not constant. As current goes up, V/I increases and the resistance therefore gets bigger. This is because filaments are very narrow and so heat up appreciably as current increases.

We can still apply $V = IR$ but only at a given voltage and current - i.e. if we double our value of voltage the current will increase by less than double because the filament's resistance has increased. So if the resistance of a filament bulb is 6Ω at 12V it will be less than this at a lower voltage; it will <u>not</u> be constant.

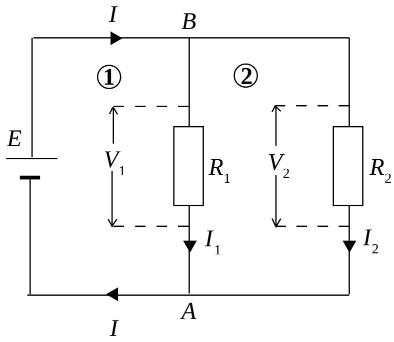
3. Diode



A diode only lets current through when it is flowing in one direction. When the diode is facing the direction of current flow it is said to be in **forward bias**. Even in forward bias it will not conduct until a voltage of around 0.6V is across it; this is called the "switch on" voltage. After the switch-on voltage is reached, resistance decreases as current rises. When the diode opposes the attempted direction of current flow then it is in **reverse bias** and has a very high (effectively infinite) resistance, allowing no current to flow.

We are now in a position to tackle questions based on electrical circuits. Firstly we will discuss different types of circuit and apply what we covered so far. The rest of this Factsheet is merely application of what we have learnt so far.

Parallel circuits



The emf present in the circuit supplies each charge with the same amount of energy. Using conventional current, charges flow through the circuit until they reach B where they choose loop ① or ②. They then proceed to travel through R_1 or R_2 and meet at point A to travel to the negative terminal of the cell.

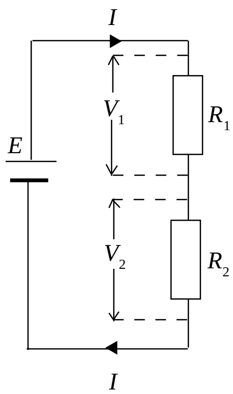
If a charge follows loop ① then it must lose an equal amount of energy in R_1 as it gained from the cell. Therefore $V_1 = E$. We can justify this by applying Kirchoff 's Second Law.

If a charge follows loop ② then the only place it can lose the energy it gained is R_2 therefore $V_2 = E$ – again a consequence of Kirchoff 's Second Law.

As charge does not disappear, the current travelling from the cell will be shared between the two pathways, so $I = I_1 + I_2$. This is justifiable using Kirchoff's First Law.

Parallel components have the **same p.d.** across them but **shared current** through them.

Series circuits

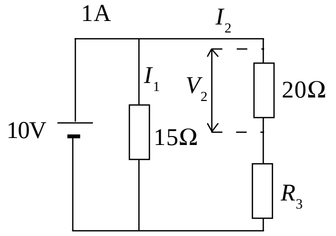


The current through both components will be the same, as every charge only has one pathway to follow. We can justify this using Kirchoff's First Law.

As charges travel through both resistances then they will give up energy. As they must give up the same amount of energy the cell gave them, $E = V_1 + V_2$. If we consider the circuit loop, we can apply Kirchoff's Second Law as justification.

Series components have the **same current** through them but **shared** p.d. across them.

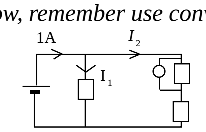
Typical Exam Question



a) Mark on the diagram above the direction of all the labelled currents. [1]

b) Using the numerical values given find values for I_1, V_2 and R_3. [5]

a) Answer marked below, remember use conventional current positive to negative. ✓



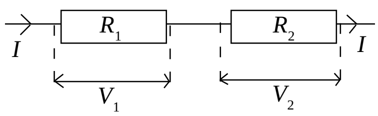
b) I_1*: Using V = IR*
$I_1 = 10/15 = 0.67A$ ✓
V_2*: Firstly we need to find the current and then we can apply V=IR.*
We find I_2 *using Kirchoff's first law:* $I = I_1 + I_2$.
So $I_2 = I - I_1 = 1 - 0.67 = 0.33A$ ✓
Now we can apply V=IR
$V_2 = 0.33 \times 20 = 6.6V$ ✓
R_3*: Now we need to find the voltage across* R_3 *using Kirchoff's second law.*
$V_3 = 10 - 6.6 = 3.4V$ ✓
And again using V = IR
$R_3 = 3.4/0.33 \approx 10\Omega$ ✓

Exam hint: *A lot of students realise they have to use V =IR but then apply it wrongly. E.g. in the above question we have been very careful to use the value of V and I that applies to the component we are dealing with. When calculating* R_3 *we made sure we were using the voltage across* R_3 *(3.3V not 10V) only and the current through it (0.33A not 1A).*

Total effective resistance

When components with resistance are placed in series or parallel then we can derive a formula to find their total effective resistance. In other words the value of the one resistor that could replace them to give the same resistance as they are both giving together.

Resistance in series



The total p.d. is shared so that: $V = V_1 + V_2$ (Kirchoff 2).

Using $V = IR$ and I is the same in both $V = IR_1 + IR_2$ (Kirchoff 1).

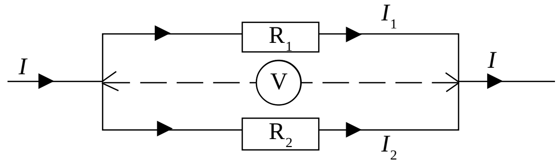
Then $V = IR_{eff}$ where R_{eff} is the total effective resistance of the two resistors.
This gives: $IR_{eff} = IR_1 + IR_2$
Cancelling the I's gives $\boldsymbol{R_{eff} = R_1 + R_2}$

This tells us that even a very small resistance placed in series with another resistance increases the circuit resistance. The current now has to travel through the original resistor and a new resistance and so finds it harder to flow. **A resistor in series always increases the effective resistance.**

Ammeters are always connected in series with the component we are interested in, so that the same current passes through the ammeter and the component. If they have a resistance then they will decrease the amount of current flowing through the component which will change the curcuit.

It is important ammeters have a low resistance and in the ideal case they should have zero resistance.

Resistance in parallel



We know that the current is shared by both resistors.

$I = I_1 + I_2$ (Kirchoff 1).

Using $I = V/R$ we get $I = V/R_1 + V/R_2$.

The total current = V/R_{eff} where again R_{eff} is the effective resistance of the two resistors. So using this; $V/R_{eff} = V/R_1 + V/R_2$

Cancelling the V's, as they are the all equal (Kirchoff 2)

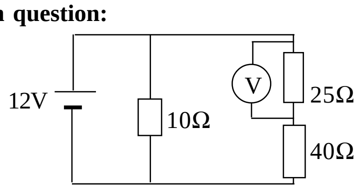
$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Now, even if we substitute a very large resistance into R_2 we still find the overall effect is to decrease the circuit resistance. This is because the current not only has the path through R_1 it had before but also a second path through R_2, therefore the resistance to its passage is reduced. **A resistor in parallel always decreases the effective resistance.**

Voltmeters are always connected in parallel. This means they will decrease the effective resistance of the component they are measuring and therefore lower the voltage across it. **It is necessary to use a voltmeter with a high resistance relative to the component it is across to gain accurate readings. Ideally a voltmeter should have an infinite resistance - this is effectively the case with a cathode ray oscilloscope.**

You may not need to learn these derivations but you should certainly understand them. These equations can be extended to more resistors.

Typical exam question:



Given the voltmeter shown in the above diagram can be taken to have an infinite resistance find:
a) the total resistance of the circuit. [3]
b) the current drawn from the cell. [2]
c) the reading on the voltmeter. [4]

*a) First work out series combination, then combine with 10Ω resistance. Work in this order as the 10Ω is in parallel with **both** 25Ω **and** 40Ω*
. $R_{eff} = R_1 + R_2 = 25 + 40 = 65\Omega$ ✓
Parallel combination:

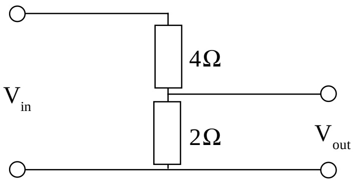
$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_{eff}} = \frac{1}{65} + \frac{1}{10} \Rightarrow R_{eff} = 8.7\Omega$$ ✓ ✓

b) $I = V/R_{total} = 12/8.7$ ✓ $= 1.38A$ ✓
c) The current through the 25Ω resistor is the same at any point along that branch of the circuit. $I = 12/65$ ✓ $= 0.18A$ ✓
Now we have the current through the 25Ω resistor and using its resistance. $V = IR = 0.18 \times 25$ ✓ $= 4.5V$ ✓

Potential dividers

We looked at series circuits earlier and saw how their components share the voltage applied to the circuit. One device which makes use of this is the **potential divider**. (Fig 1)

Fig 1. Potential divider



Potential dividers can supply a p.d. of any value up to the value of the supply p.d. by varying the size and arrangement of the resistors. This means we can tap off different p.d.'s by placing our components across one of the resistors being supplied by the fixed source.

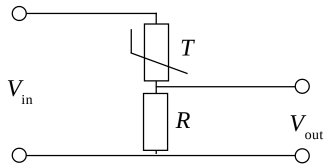
Potential dividers make use of the fact that the amount of p.d. across resistances in series is proportional to their resistance; $V \propto R$ due to Ohm's law. *I* is constant because the resistances are in series and therefore current is the same in both. So if we know the proportion one of the resistances is relative to the resistance of the whole circuit then we can work out the fraction of the p.d. it will take. This is easier to grasp if you consider the circuit above: The potential difference is shared across the 2Ω and 4Ω resistors. The total resistance is 6Ω so the 4Ω resistor will take 4/6 or 2/3 of the supply voltage (V_{in}), and the 2Ω will take 2/6 or 1/3 of V_{in}.

If we reconsider the previous exam question, we were asked to find the p.d. across a 25Ω resistor when it was connected in a series combination with a 40Ω resistor, the combination having 12V across it.

Now we can say the resistor is 25/65 = 5/13 of the total resistance and so will take $5/13 \times 12V = 4.62V$ - in agreement with our previous answer which had been rounded. Using this approach is not compulsory but it can speed up calculations or allow you to check your answers.

Thermistors

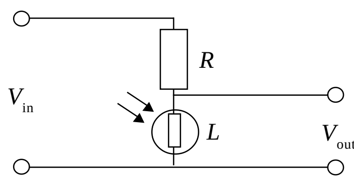
A thermistor can be used in a potential divider to provide a device that responds to temperature variation.



As temperature increases then the resistance of *T* falls. This means *R* takes a greater proportion of the voltage drop. If a fan were connected to V_{out} as the temperature increased, it would receive a greater and greater voltage.

Light Dependent Resistor

In the LDR circuit below as the light levels increase then the resistance of the LDR decreases and so does the p.d. across it.

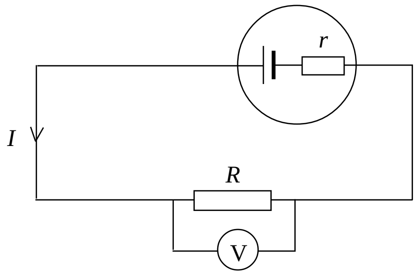


If a lamp were placed across V_{out} then it would be illuminated when dark and switch off when light.

Internal resistance

A cell itself has some resistance - known as **internal resistance**. A electron leaving the negative terminal will be involved in collisions within the cell itself and will encounter resistance. Some of the energy that the emf supplies to the moving charge is used in overcoming the internal resistance rather than in the rest of the circuit itself and so the circuit receives a lower voltage than it would do if there was no internal resistance. The voltage in the external circuit is less than the emf of the battery and as the current gets bigger, this effect becomes more pronounced. The energy dissipated in the cell due to the internal resistance is what makes it become hot when in use. We represent the internal resistance with a resistor in series with the battery labelled r (Fig 2).

Fig 2. Internal resistance



Obviously we can not get inside the cell to measure r directly so we have to use other means of finding how big it is.

Notice firstly that the current will be the same in both resistances as they are in series. Also, by Kirchoff's second law, the emf will be shared across *r* and *R*. So as $V = IR$ then: $E = IR + Ir$

and $E = I(R + r)$

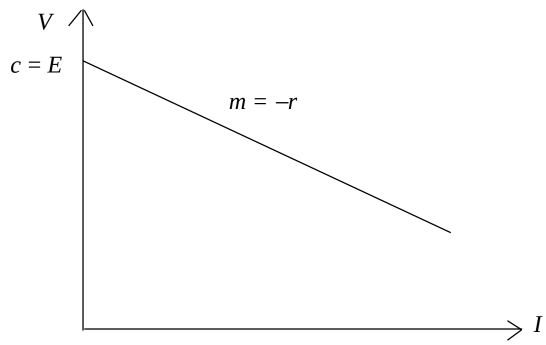
Ir is referred to as the **lost volts** because it is not available to the rest of the circuit. *IR* is the **terminal p.d.** as this is the voltage across the terminals of the cell when a current is being drawn. As *IR* is also the p.d. we measure across *R* we give it the symbol *V*. So we get

$$V = E - Ir.$$

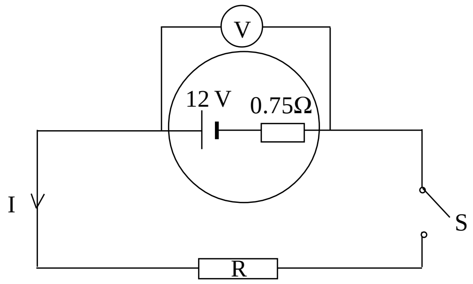
> The emf is the p.d. across the terminals of a battery (or cell) when no current is allowed to flow from it.

We could measure the emf directly with an oscilloscope, as they have effectively infinite resistance; failing this we can employ a graphical approach. If we replace *R* with a variable resistor we can obtain a set of values for *V* and *I* using an appropriately placed voltmeter and ammeter. This gives us *V* as our *y* variable and *I* as our *x* variable.

Compared with y = mx + c we get a straight line for a plot of V vs I with a gradient of -r and a y-intercept of E.



Typical Exam Question



With the switch S open the voltmeter, which has a very high resistance, reads 12V. After it is closed the reading falls to 9.5V and then rises shortly afterwards to 10V. Given the 10Ω resistor in the diagram is initially at 0°C answer the following:

a) Why does the voltmeter read 12V with S open? [2]
b) What is the value of R initially? [3]
c) What is the value of the current after a short time [1]
d) What has happened to the value of the R? Explain why. [2]

a) *It reads 12V as this is the emf ✓and the emf is present across the terminals when no current flows ✓*

b) *To find the value of R we need the current through it, we can find the current through r and, as they are in series, use this to find R.*
V = 12 – 9.5 = 2.5V
I = 2.5/0.75 = 3.33A ✓
Now use: E = I(R + r) ✓ ⇒ 12 = 3.33(R + 0.75)
R = 2.85Ω ✓

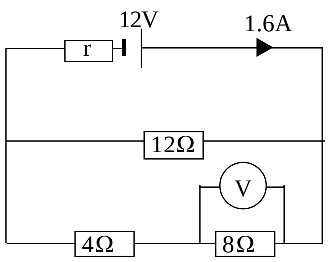
c) *V = 12 – 10 = 2V*
I = 2/0.75 = 2.67A ✓

d) *If I has decreased then R must have increased ✓ this is because it has increased in temperature ✓as it started at 0°C and the resistance of a resistor increases with temperature.*

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A circuit is set up as below. The cell does <u>not</u> have negligible internal resistance. For the first part of the question you can assume that the voltmeter has negligible resistance.



a) (i) What is meant by the internal resistance of a cell? Explain the effect it has on the terminal p.d. of the cell compared to its emf. [2]

The resistance the current inside has to overcome before it leaves the cell.✓It lowers it. ✗

Student has not explained the effect although the statement is correct.

a) (ii) What is the value of the internal resistance? [3]

E = I(R + r)✓
1/R = 1/12 + ¼ + 1/8 ∴ R = 2.18Ω ✗
12 = 1.6(2.18 + r) r = 5.32Ω ✗

Even though the student's method is essentially correct they have misunderstood the arrangement of the resistors by trying to combine all the resistances in one equation

The voltmeter is now replaced with another one having a resistance of 20Ω.
b) (i) What is the total resistance of the circuit now? [3]

1/R = 1/20 + 1/8 R=5.7Ω✓
R = 5.71 + 4 = 9.71Ω
R = 9.71 + 5.32 = 15.03Ω ✗
1/R = 1/15.03 + 1/12 = 6.67Ω ✗

The student was correct until s/he mistakenly assumed r was in series with three resistors in the bottom branch of the circuit – not true. In fact r is in series with these three and the 12Ω, so the correct course of action was to use the parallel formula and then add in r. Note the student would not have been penalised for using 5.32Ω. S/he would have received error carried forward marks.

b) (ii) Find the reading on the voltmeter. [4]
V= IR
V = 1.6 x 5.17 = 9.13V ✗✗✗✗

The student has used the correct resistance but the current is wrong for two reasons. The current splits up and so it does not flow through the combination and, as the voltmeter's resistance has changed then the current will have changed too. Notice that the question is worth four marks – it is highly unlikely an answer this short would gain so many marks.

Examiner's answers

a) (i) *The resistance a current must overcome within the cell ✓*
The terminal p.d. is lower than the emf as the current loses some energy in overcoming the internal resistance ✓

a) (ii) *E = I(R + r) ✓*
1/R = 1/12 + 1/(4 + 8) ∴R = 6Ω ✓
12 = 1.6(6 + r) r = 1.5 Ω ✓

b) (i) *1/R = 1/20 + 1/8 R = 5.71Ω ✓ For voltmeter and 8Ω*
R = 5.71 + 4 = 9.71Ω Now the 4Ω
1/R = 1/9.71 + 1/12 R = 5.37Ω Now 12Ω
R = 5.37 + 1.5 =6.87 Ω ✓ Now r

b) (ii) *We have divided total voltage by total resistance to find the total current which must all pass through r.*
I = 12/6.87 = 1.75AΩ ✓
V = 1.75 × 1.5 = 2.62 V ✓

Now the voltage across the series combination must be
V = 12 – 2.62 = 9.38V

The voltmeter and the resistor it is across have a resistance of 5.71Ω from a total of 5.71 + 4 =9.71Ω for that branch.
(5.71/9.71) × 9.38 = 5.52V ✓✓

Questions

1. (a) Explain the following terms below:

(i) internal resistance; [1]

(ii) "lost volts"; [1]

(iii) terminal p.d.; [1]

(iv) electromotive force; [1]

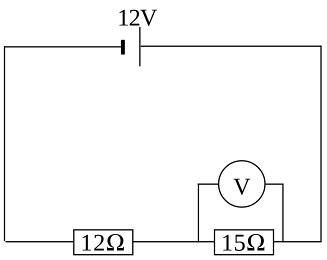
(b) State an equation that relates emf, currrent and internal resistance [2]

(c) (i) Under what conditions are the emf and terminal p.d. equal? [1]

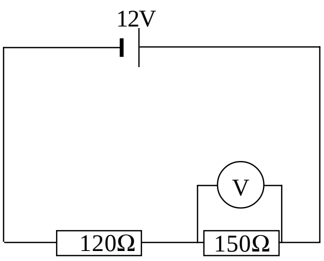
(ii) Which fundamental laws are Kirchoff's 1st and 2nd laws based upon? [2]

(d) Two circuits are shown below. The resistance of the voltmeter is 200Ω. You may assume the cell has negligible internal resistance.

A



B



(i) Find the reading on the voltmeter in **A**. [3]

(ii) Find the reading on the voltmeter in **B**. [2]

(iii) What would be the p.d. across the 15Ω resistor in **A** and the 150Ω resistor in **B** if the voltmeter were not present? [2]

(iv) Explain why the voltmeter is much more appropriate to use in circuit **A** compared with circuit **B**. [2]

Answers

1. (a) (i) The internal resistance is the opposition to current flow charge experiences inside a cell. ✓

(ii) The "lost volts" are the voltage across the internal resistance. (So called because as they are not available to the circuit). ✓

(iii) Terminal p.d. is the p.d. across the part of the circuit external to the cell. ✓

(iv) Emf is the total voltage the cell supplies to the external circuit and the internal resistance. ✓

(b) $E = I(R + r)$ ✓ where E = emf, I =current, R = external resistance and r = internal resistance ✓

(c) (i) The emf and terminal p.d. are equal when no current is drawn from the cell. ✓

(ii) Kirchoff's 1st Law: conservation of charge. ✓
Kirchoff's 2nd Law: conservation of energy. ✓

(d) (i) 1/R = 1/15 + 1/200 R = 13.95Ω ✓
(13.95/(13.95 + 12)) × 12 ✓ = 6.45V ✓
For V and 15Ω using potential divider idea

(ii) 1/R = 1/150 + 1/200
R=85.7 Ω ✓
(85.7/ (85.7 + 120) x 12) = 5V ✓

(iii) (15/(12 + 15)) × 12 = 6.67V ✓ for *A*
(150/(120 + 150)) × 12 = 6.67 ✓ for *B*

(iv) The answer for circuit *A* is much closer to 6.67 than that of circuit *B* because the voltmeter resistance is significantly larger than 12Ω or 15Ω. ✓
In *B* the voltmeter resistance is comparable with 120Ω or 120Ω or 150Ω and will give an inaccurate reading. ✓

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